

Writing
ARITHMETICK
and
Merchants Accounts
or the true Italian method of
BOOK-KEEPING
Taught by,
William Webster
in Orange Court near
LEICESTER SQUARE
London,

Youth Boarded.

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A
COMPENDIOUS COURSE
OF
Practical Mathematicks.
Particularly Adapted to the
Use of the GENTLEMEN
OF THE
ARMY and NAVY.

IN THREE VOLUMES.

For the most Part translated from the TRACTS publish'd
in *French* by P. HOSTE, Professor of MATHEMATICKS
in the *Royal Academy* of Thoulon.

By WILLIAM WEBSTER.

VOL. I. containing,
ARITHMETICK
VULGAR and DECIMAL.

L O N D O N :

Printed for A. BETTESWORTH in *Pater-Noster-Row*,
and C. KING in *Westminster-Hall*. M.DCC.XXX,

COMPREHENSIVE COURSE

OF

Practical Mathematics

Adapted to the

USE of the GENTLEMEN

OF THE

ARMY AND NAVY

By

WILLIAM GREGG, Esq.

Author of

THE

ART of

NAVIGATION

LONDON

Printed by

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ARITHMETICK
IN
EPITOME:
OR, A
COMPENDIUM
Of all its
RULES,
BOTH
VULGAR *and* DECIMAL.

In Two PARTS.

To which are now Added,
Clear and plain DEMONSTRATIONS
deduc'd from the Principles of *Arithmetick*
itself; without either Reference to *Euclid*,
or Use of *Algebra*.

By W. WEBSTER, Writing-Master.

The FOURTH EDITION, *carefully Corrected.*

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M.DCC.XXIX.

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To the RIGHT HONOURABLE

Sir Robert Walpole,

Knight of the Most Noble Order
of the Garter, First Lord Commissioner
of the Treasury, and Chancellor
of the Exchequer, &c.

S I R,



HO' the distrust so natural to
a young Author on his first
Performance, did indeed dis-
courage me from complying
with my inclination, in offering this
Attempt to your HONOUR on its first
publication; yet being a little encou-
raged by the favourable reception its
first Edition met with in the World, I
did, with all humility, presume to lay
the second Impression at your Feet;
well knowing the advantage it would

A 2

appear

DEDICATION.

appear with under the Patronage of a NAME so eminent for the encouragement of *Clerkship*, and of every other part of useful *Literature*.

Upon the third revifal of this Work for the prefs, I made fuch Additions and Improvements, as have, I hope, given it fomewhat a better Title to fo great a Protection: but ftill its Imperfections, I am very fenfible, will be but too apparent to your HONOUR's penetration; yet that Goodnefs, of which I have already had fo abundant experience, gives me hopes your HONOUR will both excufe what may remain amifs, and forgive this renew'd prefumption of

Your HONOUR's

Moft Obedient, and

Moft Humble Servant,

William Webfter.



T H E
P R E F A C E.

THE Indulgence with which the
former Editions of this Treatise
have been received, has encourag'd
me to endeavour the making it still
more worthy the acceptance of the Publick;
not only by correcting the many Errors
which had before escaped, but also by en-
larging it with very considerable Additions:
having, in this Impression, added the *Theory*
to the *Practick Part*, and endeavour'd to
inform the Reader's judgment, as well as
to assist his memory.

But this I have attempted in a manner
very different from the ordinary practice of
Writers on this Subject, having never once
quoted *Euclid*, nor perplex'd the Learner

The *P R E F A C E*.

with (to him unintelligible) *Algebraical* Demonstrations.

ARITHMETICK and GEOMETRY, are justly look'd upon as the two main pillars of the Mathematicks, upon which the whole Fabrick is rais'd ; but in comparing them, they are too often confounded: for how great a similitude soever there may be betwixt them, in some respects ; yet, as their Subjects are different, so their Principles are certainly distinct, and each may be accounted for, without references to the other. Such comparisons may indeed be useful by way of Illustration, and give great satisfaction to those who have a competent knowledge in both : But conclusions from *Geometrical* Demonstrations, are beyond the reach of the young Arithmetician ; and the Theory of *Lines* and *Diagrams*, as they are commonly made use of, can only serve to bewilder and confound him.

ALGEBRA, indeed, is not properly a Science distinct from Arithmetick, but is only a different method of Computation, perform'd by substituting *Letters* in the place of *Figures*, and expressing the several parts of the operation by *Symbolical Characters*. 'Tis true, great discoveries have been made in Numbers by this means, and Arithmetick

The P R E F A C E.

Arithmetick owes many of its improvements to this admirable invention: besides, it furnishes us with a short way of arguing, and brings the proofs of a Proposition into a little compass. But however useful it may be to those who understand it, it is certainly an unintelligible jargon to the mere Numerist, and can give him very little satisfaction when laid before him as Demonstration.

Our knowledge in science should rise gradually, from one step to another, and our references should be always backwards to what went before; but as the useful parts of Arithmetick have been commonly taught by practical Rules, which are usually taken upon content, so little exactness is generally observ'd in the order of their delivery, as to their dependance upon each other: the reason of which is plain, for as all Affairs do not require the whole knowledge of Numbers, and as most common Business may be done without much skill in *Fractions*, so it is not only common, but reasonable enough, to carry the Learner thro' whatever may be perform'd by *Whole-Numbers*, before he is enter'd upon the greater difficulties of *Fractional-Parts*. However, the gradation mentioned, should be observed as much as possible, especially in the business of Demonstration, and the Maxims referred to for explanation,

The P R E F A C E.

planation, should never be of a more abstruse nature than the thing to be explain'd.

I have therefore endeavour'd to give the inquisitive Reader who would understand the reason of things, all the satisfaction I can, and have attempted to account for the Principles of Arithmetick, from such self-evident Propositions, and natural considerations, as flow most immediately from Numbers themselves, and fall within the comprehension of such who are the common consulters of *Arithmetical Tracts*; and who, it may be presum'd, are generally unacquainted with the more abstruse parts of the Mathematicks.

My first design in this Treatise was, according to its Title, to epitomize the Art; and to reduce to a *pocket Size*, what has sometimes been swell'd to a *folio*. Indeed what I first published, was no more than a practical Memorandum, and consequently consisted only of Rules and Examples, with few or no explications: but tho' the enlargements are now very considerable, I have not yet gone beyond my design; it is still a Compendium, and far short in bulk of other Treatises on the subject. I may therefore now recommend it to the ingenious *Clerks*, and *Accomptants* of the several

The P R E F A C E.

ral Offices of Great Britain (for whose use especially it was at first design'd) and to such other Gentlemen whose Business requires the practice of Arithmetick, with more confidence than before, it being very little increased in Size, tho' very much improv'd in Precept.

As I suppose my Reader acquainted with the manner of performing the common Rules of *Addition*, *Subtraction*, *Multiplication*, and *Division*; so I have made the Work short, by omitting such needless instructions: but the *Reasons* of those operations (being the Fundamentals of Arithmetick) I thought necessary to lay down. The *Rule of Three* I have also endeavour'd to explain, that the application of *Proportion*, as it runs thro' almost all the other Rules, may be fully understood.

And for those practices of Arithmetick which will admit of no other than an *Algebraical* Demonstration, being generally parts more curious than useful, such as are some propositions in *Progression*, and the *Double Rule of False*; I have thought it sufficient to give the practical Rules for their performance, without their Demonstration; since those who are proficient in *Algebra*, cannot be thought to want such explanation;

The PREFACE.

tion, and those who are not, cannot be supposed to understand it.

I have indeed ventured to account for the Extraction of the Roots, in a manner which seems something contradictory to my profess'd design, but when it is considered that the Subject of those operations is entirely Geometrical, though the Operations themselves are numerically perform'd, it will be found that there is no other rational method of Demonstration, but that of *Lines*; which, however, I hope, I have so laid down, as to be clear and intelligible to every Reader of common sense, who but understands the difference betwixt a *Line*, a *Surface*, and a *Solid*.

As to the disposition of the *Chapters*, they follow much in the same order as they are generally taught in *Schools*: for tho' I am sensible that the Rules of *Practice* are not thoroughly to be understood without some knowledge of *Vulgar Fractions*, yet I have chose to place Fractions in the last Chapter of *Vulgar Arithmetick*, because (as I before observ'd) most common Business may be done by Whole Numbers. So the speculative Rules of *Progression*, or *Arithmetical* and *Geometrical Proportion*, which in a just *Theoretick* way of treating the subject

The P R E F A C E.

of Numbers, ought to precede the *Rule of Three*, I have thought better to place just before *Vulgar Fractions*: however, *Progression* and *Vulgar Fractions*, being each distinct Chapters, the Reader may look into them whenever he thinks it most proper.

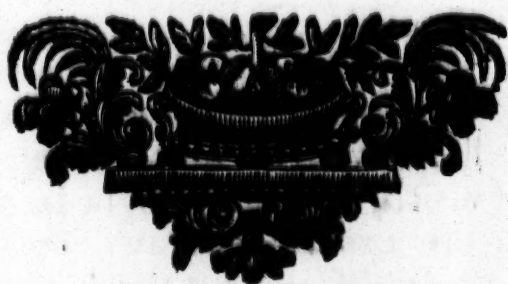
The Second Part of this Treatise which contains the Doctrine of *Decimal Fractions*, &c. is a kind of Arithmetick peculiarly, as it were, adapted to the concerns of *Gentlemen*: I have therefore been more large than ordinary upon that Subject, and have run over the several Rules again, to shew its particular use and application. In the Chapter of *Interest* and *Rebate*, its excellency will most appear, every Calculation of that kind being perform'd to great advantage by decimal Numbers. Under that head will likewise be found several useful *Tables* (which, that they might be perfectly correct, are exactly engrav'd on Copper) for the ready discovering the Amount, and present Worth both of single Sums, and of Annuities. There are, besides these, some other necessary *Tables* interspers'd through this part of Arithmetick (which are also with the same exactness, printed from copper Plates) as those for the reduction of the known parts of Coin, Time, Weight, and Measure, into decimal Parts of their

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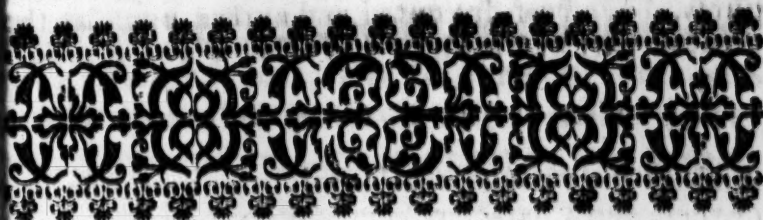
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The *P R E F A C E*.

proper *Integers*; and also that most useful *Table*, which shews the number of Days from any day in any Month, to the same day in any other. The *Contents* will shew where to find any of these things, and in the Work itself, are sufficient explanations both of their formation and use. I shall therefore add nothing further by Way of Preface, but submit the whole Performance to the judgment and candour of the Publick.



VULGAR



VULGAR ARITHMETICK,

I N

Whole Numbers and Fractions.

P A R T I.



S it is inconsistent with the intended brevity of this Treatise, to enter upon a nice enquiry after the first Inventors of Arithmetick, or by what degrees it has been rais'd to its present perfection: To neither is it my design, in the following pages, to initiate absolute Beginners; but supposing my Reader acquainted, at least, with the methods of Adding, Subtracting, Multiplying, and Dividing, (tho' perhaps ignorant of the Reasons thereof) I shall lay before him, in as few words as possible, the Reasons of those Fundamentals, and their Application throughout all the varieties of practice.

Arithmetick (which is justly defined *The Art of Reckoning*) has for its subject Number; and teaches us to give proper answers to all such questions as demand *How many?* For the more ready performing
of

[2]

of which, the ten following Characters, or Figures, have been invented and agreed upon by the greatest part of the known World, *viz.*

1 2 3 4 5 6 7 8 9 0.

And by the various combinations and repetitions of these ten Characters, may every Number, how great soever, be easily and expeditiously express'd.

Thus much in general: I now proceed to particulars.



CHAP.

C H A P. I.

*Of the five first and fundamental Rules,
viz. Numeration, Addition, Subtraction,
Multiplication, and Division.*

NUMERATION, OR NOTATION.

BY Numeration, we learn the different value of Figures, by their different places ; and, of consequence, to read or write any Sum, or Number.

The TABLE.

9	Unites.	1
90	Tens.	12
900	Hundreds.	123
9000	Thousands.	1234
90000	X Thousands.	12345
900000	C Thousands.	123456
9000000	Millions.	1234567
90000000	X Millions.	12345678
900000000	C Millions.	123456789

From this Table may be observ'd :

1. The names of the several places, *viz.* Unites, Tens, Hundreds, &c. which proceed (encreasing by a ten-fold proportion) from the right hand to the left.

2. That

2. That every Figure hath two values, one in itself; the other, from the place it stands in. Thus, on the left side of the Table, the figure 9 in the upper line, standing in the Unites place, is only nine; but in the second line, being removed into the place of Tens, becomes ninety; and in the third line is nine hundred, &c.

3. That tho' a Cypher is nothing in itself, yet it gives value to other figures, by removing them into higher places.

All which being very obvious, I proceed to the next Rule.

A D D I T I O N.

BY Addition, we find the whole, or total, of two or more parts, or sums.

In setting down the numbers to be added, care must be taken to place every figure in its proper column; that is, Unites under Unites, Tens under Tens, &c. Then will the reason of the work (the manner of which, I suppose my Reader well acquainted with) appear very evident from this undeniable Maxim, *viz. That the Whole is equal to all its Parts.* And the method of setting down the total may easily be accounted for, from the nature of Numeration, which explains the different value of places, as they proceed from the right to the left hand: For as 9 is the greatest simple character, or figure; so every number exceeding 9, being compound, must require more places than one to express it. Thus the number 10 can no otherwise be express'd in figures, but by removing the figure 1 into the place of Tens, which is done by supplying the Unites place with a cypher: And as it is the same with every other column (Ten being still the Proportion of increase)

con-

consequently, when the sum of any column amounts to 10, or more, the unites exceeding, if there be any, or a cypher, if none, must be set under such column; and the ten, or tens, of the amount carry'd on as so many unites, to the next column on the left.

What is here observ'd, as to carrying the Tens (the proportion of increase) from one column to another in Integers, may be as justly apply'd to the numbers we stop at, in adding sums of different Denominations.

For your greater ease in casting up of Money, learn the following Table :

PENCE-TABLE.

Pence.	s.	d.	Pence.	s.	d.
20	1	8	80	6	8
30	2	6	90	7	6
40	3	4	100	8	4
50	4	2	110	9	2
60	5	0	120	10	0
70	5	10			

Examples in Whole Numbers, and Money:

l.	Yards.	l.	s.	d.	
756	1325	735	18	09	$\frac{1}{4}$
132	4532	423	10	10	$\frac{1}{2}$
458	7345	784	12	05	$\frac{1}{4}$
736	1298	297	08	04	
857	8473	542	11	11	$\frac{1}{2}$
241	5249	298	14	07	$\frac{1}{4}$
3180	28222	3082	17	00	$\frac{1}{4}$ Total.

Examples

Examples in Avoirdupoise and Troy-Weights.

AVOIRDUPOISE.

<i>Tons.</i>	<i>C.</i>	<i>q.</i>	<i>lb.</i>	<i>oz.</i>	<i>drams.</i>
753	19	3	27	15	15
347	08	1	17	10	06
283	11	0	12	03	10
549	05	2	13	09	07
251	13	0	15	03	11

2185	18	1	02	11	01	<i>Total.</i>
------	----	---	----	----	----	---------------

TROY-WEIGHT.

<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>grs.</i>
327	11	19	23
429	10	17	19
274	08	13	09
478	10	12	05
326	09	18	11

1838	04	01	19	<i>Total.</i>
------	----	----	----	---------------

1510	04	01	20	} <i>Tot. without the Top-line.</i>

1838	04	01	19	<i>Proof.</i>
------	----	----	----	---------------

To know how many Drams make an Ounce, or Grains a Penny-weight, &c. see the following Tables, Page 19.

Proof of Addition.

The proof of this Rule is usually by a second addition, without the top-line; which second sum, if, when added to the uppermost line, it makes the first total, the work is supposed right. See the last Example.

Bu

But it is as good, if not a better Proof, to produce the same total, by adding your columns both up and down.

SUBTRACTION.

Substraction teaches us, by taking a lesser number from a greater, to find the Remainder.

The reason of this Rule is evident from the same principles as Addition; Substraction being but the reverse thereof: And the number borrowed in any column (like what we stop at in Addition) being always so many as would make 1 in the next.

Examples in Integers, and Money.

<i>Yards.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
7146325	<i>Lent</i>	812	13	08 $\frac{1}{2}$
1485972	<i>Paid</i>	150	19	10 $\frac{1}{2}$
<u>Rem. 5662353</u>	<i>Rem.</i>	<u>621</u>	<u>13</u>	<u>09 $\frac{1}{2}$</u>
<u>Proof. 7146325</u>	<i>Proof</i>	<u>812</u>	<u>13</u>	<u>08 $\frac{1}{2}$</u>

	<i>l.</i>	<i>s.</i>	<i>d.</i>
<i>Borrow'd</i>	429	11	08 $\frac{1}{2}$
<i>at several</i>	212	17	10
<i>Times.</i>	356	17	05 $\frac{1}{2}$
<i>Borrow'd in all</i>	<u>999</u>	<u>07</u>	<u>00 $\frac{1}{2}$</u>
<i>Paid</i>	<u>519</u>	<u>18</u>	<u>09 $\frac{1}{2}$</u>
<i>Rem.</i>	<u>479</u>	<u>08</u>	<u>02 $\frac{1}{2}$</u>
<i>Proof</i>	<u>999</u>	<u>07</u>	<u>00 $\frac{1}{2}$</u>

AVOIR.

AVOIRDUPOISE.

Tons.	C.	q.	lb.	oz.	drams.	
92	10	0	07	03	13	
14	13	3	11	12	05	
77	16	0	23	07	08	Rem.
92	10	0	07	03	13	Proof.

TROY-WEIGHT.

	l.	oz.	dwt.	grs.	
From	672	10	05	09	
Take	139	11	05	21	
	532	10	19	12	Rem.
	672	10	05	09	Proof.

Proof of Subtraction:

Sums in this Rule are easily proved, by adding their remainders to their lesser numbers, which (if right) will make the greater.

MULTIPLICATION.

Multiplication (which is the fourth Rule) serves instead of many Additions, the product of a Multiplication being only the repetition of the Multiplicand so many times as there are unites in the Multiplier.

Note, The ready performance of this and the next Rule, entirely depends upon the perfect knowledge of the following Table.

The TABLE.

3 times	{	3	9	7 times	{	7	49
		4	12			8	56
		5	15			9	63
		6	18				
		7	21			8	64
		8	24			9	72
		9	27				
4 times	{	4	16	9 times	{	9	81
		5	20				
		6	24			2	24
		7	28			3	36
		8	32			4	48
		9	36			5	60
5 times	{	5	25	12 times	{	6	72
		6	30			7	84
		7	35			8	96
		8	40			9	108
		9	45			10	120
						11	132
6 times	{	6	36			12	144
		7	42				
		8	48				
		9	54				

Three terms are used in Multiplication, viz. *Multiplicand*, *Multiplier*, and *Product*.
The following Example shews to which line each belongs.

[10]

EXAMPLE.

7563254138 *Multiplicand.*
23456789 *Multiplier.*

68069287242
60506033104
52942778966
45379524828
37816270690
30253016552
22689762414
15126508276

$\begin{array}{c} 1 \\ 8 \div 8 \\ 1 \end{array}$ *Proof.*

177409656468442882 *Product.*

That Multiplication serves instead of many Additions; and, consequently, that the truth of the operation depends upon the same Reasons, is easily proved: For suppose it were required to know the sum of four sevens; by Addition, the work would stand thus:

7
7
7
7
—
28

Too tedious a method for practice; being discoverable at once, by the Table of Multiplication, to be 28.

Nothing therefore remains to be accounted for in this Rule, but the reason of placing every following particular product a place nearer to the hand than the product foregoing: But regarding still had to the nature of Numeration, it plainly appears, that as we begin with the right hand figure of the Multiplier, or Units place, to the second figure standing in the place of Tens

the product thence arising must, of consequence, be ten times greater than the same figure, standing in the place of Unites, would have produced; and so on, applying the same consideration to the produce of all the other figures in the Multiplier.

Proof of Multiplication.

Multiplication is usually thus proved. Cast out the *nines* from the Multiplicand and Multiplier, and place the remainders on the right and left sides of a Cross, thus made $\perp$. These two figures multiply'd together, must have the *nines* cast out of their product, and the remainder placed at top: then casting the *nines* also out of the product of our Multiplication, place its remainder at the bottom, which, if it agrees with the figure at the top, the work is supposed right. See the Example.

A more certain Proof;

A Multiplication sum is then right, when the product, divided by the Multiplier, quotes the Multiplicand; or, divided by the Multiplicand, quotes the Multiplier.

ABBREVIATIONS.

(1st.) When either your Multiplicand, or Multiplier, or both, have one or more cyphers to the right hand, only multiply by the significant figures, and set on the right hand of the product so many cyphers as were in both the Multiplicand and Multiplier.

EXAMPLES.

527000	7583
4000	7000
2108000000	53081000

(2^{dly}.) Therefore any number may be multiply'd 10, 100, 1000, &c. only by placing on the right hand

[12]

hand of it, one, two, three, or more Cyphers. Thus, 7295 multiply'd by 10, is 72950 ; by 100, is 729500, &c.

(3dly.) To multiply any number by 5, add a cypher to it, and halve it. Or by 15, the same, and add both lines together.

EXAMPLES.

5334 multiply'd	
by 5 ——— 53340	by 15 ——— 53340
	26670
is ——— 26670	is ——— 80010

(4thly.) Any number may be multiply'd by 11, 111, or 112, &c. as in the following Examples.

EXAMPLES.

7136 multiply'd by 11.	by 111, thus: 7136
7136	7136
78496	7136
	792096
And by 112, thus: 7136	Or thus: 7136
7136	112
7136	
7136	85632
799232	7136
	799232

We now proceed to our fifth Rule.

DIV

DIVISION.

DY Division (which is the reverse of Multiplication) we find how often one number is contain'd in another.

In this rule there are also four terms, the *Divisor*, *Dividend*, *Quotient*, and *Remainder*.

EXAMPLE.

<i>Divisor.</i>	<i>Dividend.</i>	<i>Quotient.</i>
3456789	567895436783	(164284.

22221653

14809196

9820407

29068298

14139863

312707 *Remainder.*

As Multiplication serves instead of many Additions ; so Division supplies the place of many Subtractions ; as may be thus made evident. Suppose it were required to divide 28 by 7, that is, to find how often 7 is contained in 28 ; by Subtraction the work will stand thus :

28
7
 21
7
 14
7
 7
7
 0

B

Bv

[14.]

By which we find that 7 is 4 times contain'd in the number 28. But it may be discover'd by the rules of Division at one trial.

There are many methods of working this rule. The following example is divided six several ways

First.

$$\begin{array}{r} 70(2 \\ 7008(1 \\ 42645(1332 \\ 32222 \\ 333 \end{array}$$

Third.

$$\begin{array}{r} 70(2 \\ 7008(1 \\ 32)42645(1332 \end{array}$$

Second.

$$\begin{array}{r} 70(2 \\ 7008(1 \\ 32)42645(1332 \\ 32664 \\ 996 \end{array}$$

Fourth.

$$\begin{array}{r} 21 \\ \hline .8 \\ 10 \\ 10 \\ 32)42645(1332 \\ 32 \\ 96 \\ 96 \\ 64 \end{array}$$

Fifth.

$$\begin{array}{r} 32)42645(1332 \\ 32 \\ \hline 106 \\ 96 \\ \hline 104 \\ 96 \\ \hline .85 \\ 64 \\ \hline 21 \end{array}$$

Sixth.

$$\begin{array}{r} 32)42645(1332 \\ \hline 106 \\ \hline 104 \\ \hline .85 \\ \hline 21 \end{array}$$

(2dly.
hat to
o man
end, a

So
(3dly.
f one
utting
ay be

Note, The two last, call'd the *Italian* ways, are most generally used.

Mr. *Alingham* has, indeed, set down nine ways; but his three others are so very like the 1st, 2d, and 3d of these, that they can scarce be call'd different.

ABBREVIATIONS.

(1st.) If there are any cyphers on the right hand of your Divisor, you may cut off so many cyphers, or figures, on the right hand of your Dividend; but remember to bring them down (if figures) to the remainder.

EXAMPLE.

$$21 \overline{) 008645} | 29(411$$

84

24

21

35

21

1429

(2dly.) By the foregoing rule, you may observe, that to divide by 10, 100, 1000, &c. is only to cut off so many figures from the right hand of the Dividend, as there are cyphers in the Divisor.

EXAMPLE.

$$1 \overline{) 000} 43682 | 735 ($$

So the Quotient is 43682, the Remainder 735

(3dly.) When your Divisor is 12, or consists only of one single figure, or can be reduced to one, by cutting off cyphers from its right hand, the work may be easily perform'd in one line, thus:

R U L E.

Drawing a line under the Dividend, set down under its figure, how often the Divisor is contain'd in it; what remains, imagine placed before the next figure; and considering how often your Divisor is contain'd in the sum it makes, set down the number underneath, as before; and so proceeding through all the figures, set down what remains at last, in the place where your Quotient used to stand.

E X A M P L E S.

$$\begin{array}{r} 4 \overline{)93645(1} \\ \underline{23411} \end{array} \quad \begin{array}{r} 12 \overline{)83675(11} \\ \underline{6972} \end{array} \quad \begin{array}{r} 7 \overline{)005635(15(} \\ \underline{805} \end{array}$$

If you are to divide several numbers by one common Divisor (as in the calculating of Tables, &c.) that you may know exactly at once how often your Divisor will go, in some convenient corner, make a Table of your Divisor, by multiplying it severally by all the nine digits: Thus, suppose 562 your Divisor:

562	1
1124	2
1686	3
2248	4
2810	5
3372	6
3934	7
4496	8
5058	9

Proofs of Division.

(1st.) Multiplication and Division mutually prove each other: For as if you divide the Product of a Multiplication by the Multiplier, the Quotient will be the Multiplicand; so, if you multiply the Quotient

Quotient of a Division by the Divisor (taking in the remainder) the product will be the Dividend.

(2dly.) Another proof of Division is, by adding together those lines in the following example, mark'd with Asterisks, (being the particular products of the Divisor, multiply'd severally by each figure in the Quotient, together with the remainder of the Division) the total of which (if right) will be the Dividend.

(3dly.) Division may also be proved as Multiplication, by a Cross, thus; casting out the nines from the Divisor and Quotient, place the remainders on its right and left sides; then multiplying the two figures so placed together, and casting the nines from the product, add what's left to the remainder of the Division; and still casting out the nines, let the overplus be placed at the top; then also casting the nines from the Dividend, let down the figure remaining at the bottom, which if it agrees with that at top, the work may be suppos'd right. See each proof in the following examples.

E X A M P L E.

736)863256(	1172
736 *	736
1272	7032
736 *	3516
5365	8204
5152 *	862592
2136	664 Remainder.
1472 *	863256 1st Proof.
664 *	

863256 2d Proof.

Before we proceed to *Reduction*, it will be proper to insert the following Tables.

B 3

TABLES

TABLES of English Coins, Weights
and Measures.

First of COINS.

ACCOUNTS are kept in Pounds, Shillings,
Pence, and Farthings thus divided.

4 Farthings } make { 1 Penny, } thus mark'd { d.
12 Pence } { 1 Shilling, } { s.
20 Shillings } { 1 Pound, } { l.

But the usual Coins are,

			l.	s.	d.		
Of Gold,	{	A Jacobus,	{	1	5	0	
		A Carolus,		1	3	0	
		A Guinea,		1	1	0	
		A $\frac{1}{2}$ Guinea,		0	10	6	
Of Silver,	{	A Crown,	{	0	5	0	
		A $\frac{1}{2}$ Crown,		0	2	6	
		The names of the rest speak their value, as a Shilling, a Six-pence, a Groat, or 4 d. a Three-pence, a Two-pence, a Penny.					
Of Copper,	{	A $\frac{1}{2}$ Penny,	{	0	0	0 $\frac{1}{2}$	
		A Farthing,		0	0	0 $\frac{1}{4}$	

thus writ,

Besides the above-mention'd, we have still in use the names of some other pieces, which are now but imaginary, viz.

y, viz.		l.	s.	d.
A Mark,	} Value	0	13	4
An Angel,		0	10	0
A Noble,		0	6	8

WEIGHTS

WEIGHTS.

TROY.

4 Grains }
 6 Pennywts. } make { 1 Pennywt.
 2 Ounces } { 1 Ounce,
 { 1 Pound, } thus mark'd { dwts.
 { oz.
 { lb.

By *Troy-Weight* are weighed Jewels, Gold, Silver, Corn, Bread, and all Liquors.

APOTHECARIES.

6 Grains }
 2 Scruples } make { 1 Scruple,
 8 Drams } { 1 Dram,
 2 Ounces } { 1 Ounce,
 { 1 Pound, } thus mark'd { dwts.
 { oz.
 { lb.

By these *Weights*, Apothecaries compound their Medicines; but buy and sell their Drugs by *Avoirdupois*.

AVOIRDUPOISE.

6 Drams }
 6 Ounces } make { 1 Ounce,
 8 Pounds } { 1 Pound,
 4 Quarters } { 1 quarter of a Hund.
 6 Hundred } { 1 Hundred,
 { 1 Ton, } thus mark'd { dwts.
 { oz.
 { lb.
 { grs.
 { c.
 { ton

This is, at present, the common weight of England, by which, Butter, Cheese, and all Groceries, &c. are weighed.

Note, One pound *Avoirdupoise* is equal to 14 oz. 1 dwts. 15 grs. $\frac{1}{2}$ Troy; and one ounce Troy is equal to 1 oz. 1 dram, and something above $\frac{1}{2}$, *Avoirdupoise*.

WOOLWEIGHT.

7 Pounds	}	make	{	1 Clove,	}	mark'd as written.
2 Cloves				1 Stone,		
2 Stones				1 Todd,		
6 Todds and $\frac{1}{2}$				1 Wey,		
2 Weys				1 Sack,		
12 Sacks				1 Last,		

LEAD.

G.
19 $\frac{1}{2}$ makes a Fodder.

MEASURES.

WINE.

2 Pints	}	make	{	1 Quart,	}	thus mark'd	{	Qrt.
4 Quarts				1 Gallon,				Gall.
6 Gallons				1 Hogshead,				Hdd.
2 Hogsheads				1 Pipe,				Pipe.
2 Pipes				1 Tun,				Tun.

BEER and ALE.

2 Pints	}	make	{	1 Quart,	}	thus mark'd	{	Qrt.
4 Quarts				1 Gallon,				Gall.
9 Gallons				1 Firkin,				Firkin.
2 Firkins				1 Kilderkin,				Kild.
2 Kilderkins				1 Barrel,				Bar.
2 Barrels				1 Butt,				Butt.

Note, 8 Gallons make 1 Firkin of Ale.

[21]

D R Y

8 Pints	}	make	1 Gallon	}	thous mark'd	1 Gall.
2 Gallons			1 Peck,			1 Peck.
4 Pecks			1 Bushel,			1 Bush.
4 Bushels			1 Coom			1 Coom.
2 Cooms			1 Quarter,			1 qr.
5 Quarters			1 Wey,			1 Wey.
2 Weys			1 Last,			1 Last.

L O N G.

3 Barley-Corns	}	make	1 Inch,	}	mark'd as usual	
2 Inches			1 Foot,			* 1760
3 Feet			1 Yard, *			Yards
5 Yards and 1			1 Pole or Perch,			make a
10 Poles			1 Furlong,			Mile.
8 Furlongs			1 Mile,			

L A N D.

40 Square Perches	}	make	1 Rood.
4 Roods			1 Acre.

Note, That a Geometrical Pace is 5 Feet, and that there are 1056 such paces in an English Mile.

C L O T H.

4 Nails	}	make	1 Quarter,	}	mark'd	1 qr.
4 Quarters			1 Yard,			1 yd.

Note, An Ell Flemish is 3 qrs of a Yard; an Ell English is 5.

12	}	make	1 Dozen.
12 Dozen			1 Small Gross.
12 Small			1 Great Gross.



C H A P. II.

Of REDUCTION.

Reduction is but an application of Multiplication and Division: For,

First, All great names are brought into small, by multiplying with so many of the little ones as make one of the great.

Secondly, All small names are brought into great, by dividing by so many of the little ones as make one of the great.

Thirdly, To change one sort of money, or weight, &c. into another, is only to bring both into one name, and to divide the one by the other.

EXAMPLES of each Sort.

(First Sort.)

l. s. d.

In 4295 12 3 how many Farthings?
'tis multiply'd by 20 because 20 s. make 1 l. Note, The

12 s. are taken in.

85912

'tis multiply'd by 12 because 12 d. makes 1 s. And here

the 3 d. are added.

1030947

'tis multiply'd by 4 because 4 Farthings make 1 Penny

Answer. 4123788 Farthings.

(Second

[23]

(Second Sort.)

In 4123788 Farthings, how many Pounds?

4)4123788(

12)1030947(3

2|0)8591|3(

4295 l. 12 s. 3 d. fact.

We divide here by 4, 12, and 20, for the same reasons that we multiply'd by them in the first example.

(Third Sort.)

Change 45967 French Crowns, at 4 6 each, into Guineas.

s. d.

Six-pences in a French Crown. } 9

8.
21
2

45967
9

42 { Six-pences in 42)412703(9850 Guineas.
a Guinea, 378

357
336

210
210

03 Six-pences.

More EXAMPLES.

In 85647 Guineas, how many Pounds Sterling?

Answer, 89929 l. 7 s.

B 6

In

[24]

In 59463 Marks, at 13 s. 4 d. how many Pounds?

Answer, 39642 l.

How many Dollars, at 4 s. 4 d. are there in 89573 Pistoles, at 17 s. 6 d?

Answer, 361737 Dollars, and 6 d. remain.

In 82 lb. 9 oz. 10 dwt. 5 grs. how many Grains?

Answer, 476885 Grains.

In 56 Tuns, 13 C. 2 qrs. 21 lb. 7 oz. 11 Drains, how many Drains?

Answer, 32505211 Drains.

How many minutes since the birth of our Saviour, it being 1729 years?

Answer, 908762400 minutes.

How many Gallons are there in 7569 Tuns?

Answer, 1907388 Gallons.

Note, These questions revers'd, will make as many more good examples.



CHAP.

(Ex
f I gi

C H A P. III.

Of PROPORTION, or the RULE of
THREE, *Direct, Indirect, and Double.*

THIS Rule is call'd the RULE of THREE, because by three numbers given we find a fourth number sought; which, when the Proportion is direct, must always bear the same ratio, or proportion, to the third number, as the second bears to the first.

The chief difficulty of this Rule, lies in stating the questions; For your direction therefore observe, that of the three given numbers, two always contain a supposition, and the third a demand;

The number then on which the demand lies, must always be the third in your stating; of the other two, you will be sure to find one of the same quality with the said third; which being made your first, the number left, must, of consequence, fall in the second place, which second number will also always be of the same kind with the fourth, or number sought.

The question being thus stated, you must (if they are not already so) bring your first and third numbers into one name, and your second (if of several denominations) into its lowest term; then multiplying your second and third numbers together, and dividing the product by your first, the quotient will be the answer of your question, in the same denomination you left your second number. See the Examples.

E X A M P L E S.

(Ex. 1st.) How many Ounces may I buy for 8 *l*. if I give after the rate of 2 *l*. for 18 Ounces?

IF

[26]

l.
If a buy — 18 — *ozs.* what will 8 ?

2)144(

Answer.—72 ozs.

(Ex. 2d.) What must I give for 122 Bills of Holland, if I pay after the rate of 34 *l.* for 138 Bills ?

Exs. *l.* *Exs.*
If 138 cost — 34 *l.* — what will 122 ?

The Remainder 8, being Parts of a Pound, are multiplied by 20, to see what Shillings they will produce ; and so the other Remainders by 12, and 4, to bring out the odd Pence, and Farthings.

This way of valuing the Remainder, is more fully explain'd in the seventh sort of Reduction of Vulgar Fractions.

34
—
488
366
—
138)4148(30 1 11
414
—
8
20
—
138)160(1
138
—
22
12
—
138)264(1
138
—
126
4
—
138)504(3
414
—
90

(Ex. 3d.)

[27]

(Ex. 3d.) How many yards of Muffin can I buy
for 42 s. 12 d. If 2 1/2 yards come to 19 s. 6 d.?

s. d. yards. s. d.
If 19 6 buy 2 1/2 what will 42 12

$$\begin{array}{r} 2 \\ \hline 38 \end{array}$$

$$\begin{array}{r} 2 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 852 \end{array}$$

$$\begin{array}{r} 2 \\ \hline 1704 \end{array}$$

49) 8520 (218 fulls.

$$\begin{array}{r} 78 \\ \hline \end{array}$$

$$\begin{array}{r} 72 \\ \hline \end{array}$$

$$\begin{array}{r} 49 \\ \hline \end{array}$$

$$\begin{array}{r} 110 \\ \hline \end{array}$$

$$\begin{array}{r} 112 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \end{array}$$

109 yards.

P R O O F.

Of every four numbers in direct proportion, the product of the two means multiply'd into each other, will be equal to the product of the two extremes so multiply'd: therefore, multiplying your first by your fourth number found, and comparing it with the produce of your second by your third, if the products agree, the work is right. Thus, in the first example, 72 (the fourth number found) multiply'd by 2 (the first number) gives 144, equal to the produce of 8 times 18, the second and third numbers.

And from hence arises the invention of the foregoing rule, for finding the fourth number; for if the second number, multiply'd by the third, be

†

equal

equal to the first multiply'd by the fourth, it is plain that if the product of the second and third be divided by the first, the Quotient must be the said fourth number; because every Dividend must be equal to the produce of its Divisor and Quotient, the remainder (if any) being also consider'd.

Sums in this rule may also be proved by a back stating; thus reversing the third Example.

l. s.	yards.	s. d.	yards.
If 42 12 will buy 109, then	19 6 will buy	2 1	
20	39	2	
852	981	39	
2	327		
1704	1704) 4251 (2 1 yards.		
	3408		
	1843		
	2		
	1686		
	18 { Remainder of the other		
	2 { stating added.		
	1704) 1704 (1		
	... 0		

Though in the former of the foregoing proofs, the manner of working the *Rule of Three* is sufficiently accounted for, and the reason of the operation plainly enough laid down; yet as this may justly be look'd upon as the main Rule of Arithmetick, and as the same application of proportion runs thro' almost all its other branches; it may, perhaps, help to make it still more evident, to look back again upon the first example, and observe it in another light. The question there stated was, *If 2 l.—18 oz.—8 l. Now had it been, If 1 l.—18 oz.—8 l. 'tis clear,*

ear, that if 1 l . would buy 18 oz. that then 8 l . would buy 8 times as many, that is 144 oz. But then, as this is upon the supposition that 1 l . is the price of 18 oz. so if the price of the 18 oz. is doubled, that is, made 2 l . 'tis plain 144 oz. will then be twice the quantity that 8 l . will buy, and must therefore be divided by 2, to give the true answer. Again, had the supposed price of 18 oz. been 4 l . then would 8 l . have bought but a fourth part of 144 oz. viz. 36 oz. consequently, in what proportion soever the first number encreases, in the same proportion must the fourth number decrease; which is plainly effected by dividing by the first number.

Note, In the operation of the *Rule of Three*, the first and third numbers, after preparation, are no more regarded, as of any denomination; but are multiply'd and divided with, only as absolute numbers, encreasing and decreasing the middle number in just proportion.

Note also, That what has been said of the reason of working the *Rule of Three*, ought also to be consider'd in the operations of *Interest*, *Rebate*, *Fellowship*, *Exchange*, and all other applications of proportion.

We now proceed to some other practical observations.

1. If you would know at what rate you must sell out your goods by retail, so as to make a proposed gain by the whole, add the money you would win to the sum the whole goods cost you; and then state your question thus: *If the whole be sold for the sum of the Cost and Gain, what must any part?*

EXAMPLE.

Suppose I would, by the selling of 32 yards of broad Cloth, which cost me 40 l . gain 5 l . for what must I sell it *per yard*? Then,

If

[30]

yards. l. yard.
If 32 be sold for 45, what will 1 ?

1.	1
40 Cost.	— l. s. d.
5 Gain.	32) 45 (1 8 1 1/2 facit.
—	32
45 Total.	—
	13
	20

32) 260 (8
256

4
12

32) 48 (1
32

16

4

32) 64 (2
64

0

(2d.) Or if damage having happen'd to the Cloth
5 l. were to have been lost by the whole, then the
said 5 l. must have been subtracted from the cost,
and the remainder made the second number, as
before.

(3d.) If you would barter or exchange your
goods for others, first find the value of your own,
and then see what quantity of the others the sum
will purchase.

EXAMPLES

EXAMPLES.

What quantity of Pepper, at 3 s. 6 d. per lb. may
have in exchange for 426 C. of Tobacco, at 43 s.
C?

$$\begin{array}{r}
 \text{First, If } \begin{array}{c} \text{C.} \\ 1 \end{array} \begin{array}{c} \text{s.} \\ 53 \end{array} \begin{array}{c} \text{C.} \\ 426 \end{array} \\
 \hline
 \begin{array}{c} 53 \\ 1278 \\ 2130 \end{array}
 \end{array}$$

Answer, 22578 Shillings.

$$\begin{array}{r}
 \text{Then, If } \begin{array}{c} \text{s.} \\ 3 \end{array} \begin{array}{c} \text{d.} \\ 6 \end{array} \begin{array}{c} \text{lb.} \\ 1 \end{array} \begin{array}{c} \text{s.} \\ 22578 \end{array} \\
 \hline
 \begin{array}{c} 7 \\ 7)48156(6 \end{array}
 \end{array}$$

Answer, 6450 Penns?

ABBREVIATION.

The work of some statings may be much short-
ed, by dividing their first and second, or first and
third numbers, by any figure, so that nothing may
main.

EXAMPLE.

EXAMPLE.

l. *Men* *l.*
If 192/00 would pay 5/00 how many would 17664 pay

$$\begin{array}{r} 32 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 2944 \\ \hline 736 \\ \hline 5 \\ \hline 8)1696 \end{array}$$

Answer, 460 Men

The numbers of the above-stating are abbreviated by first cutting off the Cyphers from the first and second numbers, and then dividing the first and third numbers by 6 and 4.

The reasons of this abbreviation may be seen in the fifth sort of Reduction of Vulgar Fractions.

More Questions in the DIRECT RULE of THREE.

The cloathing of a Regiment of 740 men, comes to 3000 *l.* how much is that for each man?

Answer, 4 *l.* 1 *s.* 0 *d.* $\frac{1}{2}$

How long shall I be laying up 10000 *l.* if I put by $\frac{1}{2}$ a Guinea a week?

Answer, 366 years, 15 weeks, 4 days.

500 Sea-men are to have 4 *d.* $\frac{1}{2}$ per day each, what will pay them for 23 months?

Answer, 6037 *l.* 10 *s.*

What will an estate of 4000 *l.* per Annum, allow a Gentleman to spend a day?

Answer, 10 *l.* 19 *s.* 2 *d.*

A Gentleman's daily expences are 13 *s.* 7 *d.* above which, he yearly lays up 600 Nobles, at 6 *s.* 8 *d.* each

ch, what is his Estate worth per *ANNUM*?

Answer, 447 *l.* 17 *s.* 11 *d.*

A man owing 736 *l.* 10 *s.* compounds with his creditors for 7 *s.* 9 *d.* per *l.* what will that amount

Answer, 285 *l.* 7 *s.* 10 *d.* $\frac{1}{2}$

If at 5 *s.* per ell, I gain 8 *l.* per Cent. by my Cloth, what shall I gain per Cent. if I sell the ell at 6 *s.* 3 *d.*?

Answer, 10 *l.*

If Tobacco, which cost 13 *d.* per lb, be sold for 14 *d.* per lb, what is gain'd per Cent?

Answer, 38 *l.* 9 *s.* 2 *d.* $\frac{1}{4}$.

Bought 5 pieces of Holland, each containing 56 Flemish, at 3 *s.* 2 *d.* per ell, what shall I gain in the whole, if I sell it for 5 *s.* 8 *d.* per ell English?

Answer, 31 *s.* 4 *d.*

I have by me 96 lb. of Cloves, which cost me 58 *l.* at some damage having happen'd to them, I am willing to lose 8 *l.* in the whole; at what rate must I sell them per ounce?

Answer, 7 *d.* $\frac{1}{4}$.

A Merchant sends over to France 482 tons of Lead, at 4 *l.* 10 *s.* per Fother, 16 *l.* 19 $\frac{1}{2}$ *s.* what quantity of Wine, at 30 *l.* per pipe, may he expect in return?

Answer, 74 pipes, 19 gallons.

A Merchant sends to Spain 1300 pieces of Broad Cloth, each piece 47 yards, at 15 *s.* 6 *d.* per yard, to have returns from thence; the one half in Wine, at 5 *l.* per Ton; the other half in Oranges, at 3 *l.* 10 *s.* per Chest; what quantity of each will he have?

Answer, { 364 Tons, 1 Hoghead of Wine.
6764 Chests of Oranges.

Two men part, at the same time, from the same place; the one travels north 32 miles a day; the other, 36 miles a day south; how long will it be before they are 2000 miles asunder?

Answer, 29 days, 9 hours, 52 minutes.

The Indirect Rule of THREE.

IN the *Indirect Rule of Three*, the numbers are in reciprocal proportion, that is, the fourth number to be found, is to bear the same ratio to the second as the third does to the first, but in an inverted order; that is, the greater the third term is in respect to the first, the less must the fourth be in respect to the second.

This rule differs in its operation, from the *Direct*, in that, after the question is stated, and the numbers of the statings prepared, (as in the *Direct Rule*) your first and second must be multiplied together, and your third number be your Divisor. The Quotient, as before, will be the answer.

EXAMPLES.

(*Ex. 1st.*) What number of men must be employed to finish, in 12 days, what 43 men would be in 35 days about?

days	men	days.
If 35	— 43 —	12
43		
105		
140		
12)1505(5		
Answer, 125 Men.		

(*Ex. 2^d*)

(Ex. 2d.) How many yards of Stuff 3 qrs wide, will hang a Room which requires 420 yards of 5 qrs. wide P

$$\begin{array}{r}
 \text{qrs.} \quad \text{yards.} \quad \text{qrs.} \\
 \text{If } 5 \text{ ————— } 420 \text{ ————— } 3 \\
 \quad \quad \quad 5 \\
 \quad \quad \quad \hline
 \quad \quad 3 \overline{) 2100} \\
 \quad \quad \hline
 \text{Answer, } 700
 \end{array}$$

The reason of this operation will appear plain after what has been said in the Direct Rule by considering the last example. Now it is clear, that if the Stuff, being 5 quarters wide, there are 420 yards requir'd, then were the Stuff but 1 quarter wide, 5 times 420 yards, viz. 2100 yards, must be allowed; consequently, if the Stuff be 3 quarters wide, one third part of those yards will be sufficient; therefore 2100 divided by 3, will give the true answer requir'd, viz. 700 yards.

To know whether a Question belongs to the Direct or Indirect Rule of Three.

Observe, If the third number being more than the first number, requires more, or being less, requires less, 'tis *Direct*; but if the third number, being more, requires less, or being less, requires more, 'tis *Indirect*.

Or, without any regard to the distinction of *Direct* and *Indirect*; if more is required, let the lesser of the two extremes be the Divisor; if less, the greater.

More

More Questions in the Indirect Rule of Three.

If I lend A. 136 l. for 3 months, how long must I keep 42 l. of his, to requite my self?

Answer, 9 months, 2 weeks, 6 days.

If 46 Clerks in 30 days finish a piece of writing in what time would 77 Clerks accomplish the same?

Answer, 26 days, 18 hours, 19 minutes.

A Garrison, consisting of 1739 men, being besieged, have provisions only for 12 days, but being necessary they should hold out 3 weeks, how many men must be sent out?

Answer, 660 men.

The DOUBLE RULE of THREE.

Questions in this Rule have five Numbers proposed, and are generally answered by two statings; of which, one sometimes happens to be *Indirect*, as in the second Example.

EXAMPLES.

EXAMPLES.

(Ex. 1st.) The carriage of 32 hundred weight 5 miles, comes to 12 s. After the same rate, what must I pay to have 78 hundred weight carry'd 94 miles?

First, if $\begin{array}{ccc} \text{Q.} & & \text{Q.} \\ 32 & \text{---} & 12 & \text{---} & 78 \end{array}$

$\begin{array}{r} \text{12} \\ 32 \overline{) 936} \end{array}$ s. d. fact.

$\begin{array}{r} 296 \\ 288 \\ \hline \end{array}$

$\begin{array}{r} 8 \\ 12 \\ \hline \end{array}$

$\begin{array}{r} 32 \overline{) 96(3} \\ 96 \\ \hline 0 \end{array}$

Then, if $\begin{array}{ccc} \text{Miles.} & \text{s. d.} & \text{Miles.} \\ 56 & \text{---} & 29 & 3 & \text{---} & 94 \end{array}$

$\begin{array}{r} 12 \\ 351 \\ 94 \\ \hline \end{array}$

$\begin{array}{r} 1404 \\ 3159 \\ \hline \end{array}$

$\begin{array}{r} 56 \overline{) 32994} \end{array}$ (589

280

499

448

514

504

10

12(589(1

2|0|4|9(

Answer, 2 l. 9 s. 1 d.

Note, It had been the same thing, if the mile had been made the first and second numbers of the first stating; and the C. weights the first and second numbers of the last.

Note also, It may be done in one stating, thus:

$$\begin{array}{r} \text{C.} \quad \quad \quad \text{S.} \quad \quad \quad \text{C.} \\ \text{If } 32 \text{ --- } 12 \text{ --- } 78 \\ \quad 56 \text{ Miles.} \quad \quad 94 \text{ Miles.} \end{array}$$

$$\begin{array}{r} 192 \\ 160 \\ \hline 1792 \end{array}$$

$$\begin{array}{r} 312 \\ 702 \\ \hline 7332 \\ 12 \end{array}$$

$$\begin{array}{r} \text{---} \quad \text{S. d.} \\ 1792) 87984(49 \text{ } 1 \\ \quad 7168 \quad \text{or} \\ \hline \quad 16304 \quad \text{1. S. d.} \\ \quad 16128 \quad 2. 9 \text{ } 2 \end{array}$$

$$\begin{array}{r} \text{---} \\ \quad 176 \\ \quad 12 \\ \hline 1792) 2112(1 \\ \quad 1792 \\ \hline \quad 320 \end{array}$$

[39]

(Ex. 2d.) How many men must be employ'd to reap 420 Acres in 17 days, if there were requir'd 37 men to reap 54 Acres in 5 days ?

	<i>Acres.</i>	<i>Men.</i>	<i>Acres.</i>
First, If	54	37	420
			37

2940
1260

	<i>Days.</i>	<i>Men.</i>	<i>Days.</i>	
Then, If	5	287	17	54)15540(287
		5		108

17)1435(84
136

474
432

75
68

420
378

Ans^r. 84 Men.

7

42

Note, If you would work such questions of the *Double Rule of Three*, as have one of their proportions indirect, by one stating; you must multiply the third number of your stating, by that number you would otherwise have placed under your first; and your first number by that you would have placed under the third; as in the following Example.

C 2

EXAMPLE.

E X A M P L E.

Aeres.	Men.	Aeres.	
If 54	37	420	} The number of days which have relation to the 54 Aeres.
17		5	
378		2100	
54		37	
918		14700	
		6300	
		Men.	
* The number of days which have relation to the 420 Aeres.	918	77700	(84 answer, as before.
		7344	
		4260	
		3672	
		588	

Dr. Harris, in his *Lexicon Technicum*, teaches another way of ranging the five Numbers given in a *Double Rule of Three Question*; which, according to the following Rules, gives the solution, whether the proportion be direct or indirect. Take his method in his own words.

I. " Observing, that the given terms are always five, whereof three are conditional and antecedent, or suppositious, the other two demand the question, and are consequents answering some of the former antecedents; insomuch, that with the answer there will be as many consequents as antecedents, which must match one another in the same denomination exactly.

II. " For the right placing of the question and terms, the three terms of the conditional part are duly to be regarded: Let that which is the principal cause of Loss and Gain, Increase or Decrease, Action or Passion, be put in the first place, and that which betokeneth the space of time,

time, distance of place, &c. be put in the second place, and the remaining part in the third. The conditional part thus stated, the other two terms, wherein the demand lies, must be placed so under the former terms, that they may correspond one with another.

R U L E 1.

" Then, if the blank, or place sought, fall under the third term, multiply the three last terms for a Dividend, and the two first for a Divisor; and the Quotient gives the sixth term required.

R U L E 2.

" But if the blank fall under the first or second term, multiply the 1st, 2d, and 5th terms for a dividend, and the 3d and 4th for a Divisor: The Quotient gives the answer.

E X A M P L E 1.

" If 12 Rods of ditching be done by 2 men in 6 days, how many Rods shall be wrought by 8 men in 24 days?

Answer, 192.

The numbers being stated as before directed, they will stand thus, the blank falling under the third place.

<i>Men.</i>	<i>Days.</i>	<i>Rods.</i>
2	6	12
8	24	

Therefore, by the first Rule, the three last terms, viz. 12, 8, and 24, being multiply'd into each other, give 2304 for a Dividend; and the two first terms,

C 3

[48]

terms, viz. 2 and 6, multiply'd together, give 12 for a Divisor, which quotes 192, the answer.

EXAMPLE 1.

If 2 men work 12 Rods in 6 days, how many men will work 192 Rods in 24 days ?

Men	Days	Rods.
2	6	12
	24	192

Here, according to the second Rule, the first, second, and fifth terms multiply'd, give 2304, which divided by 288, the product of the third and fourth numbers, quotes 8, the answer.

More EXAMPLES.

If 400 pecks of Corn will serve 32 horses 108 days, what quantity will 500 horses eat in 20 days ?

Answer, 1157 pecks, $\frac{1}{4}$

If 56 gallons of Drink serve 25 persons 120 days ? how long will 200 gallons serve 12 persons ?

Answer, 892 days.



CHAP.

CHAP. IV.

Of PRACTICE, and TRET and TARE.

BY PRACTICE, Merchants compendiously cast up the price of their Commodities. And by **TRET** and **TARE**, deduct their Allowances.

Both these Rules will be best explain'd by examples; and both require the perfect knowledge of the following

TABLE.

Of a Pound.			Of a Shilling.			Of a Ton.			
s. d.			d.			C.			
The even Parts.	1	0—	2	1	0	2	—	$1\frac{1}{10}$	
	1	8—	1	$1\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{1}{2}$ —	$1\frac{1}{8}$	
	2	0—	2	—	$\frac{1}{6}$	4	—	$1\frac{1}{3}$	
	2	6—	3	—	$\frac{1}{4}$	5	—	$1\frac{1}{4}$	
	3	4—	4	—	$\frac{1}{3}$	10	—	$1\frac{1}{2}$	
	4	0—	6	—	$\frac{1}{2}$	Of a Hund.			
	5	0—	Of a Penny.			lb.	14	—	$1\frac{1}{8}$
6	8—	Far.			16	—	$1\frac{1}{7}$		
10	0—				Of $\frac{1}{2}$ C.				
						lb.	7	—	$1\frac{1}{8}$
						8	—	$1\frac{1}{7}$	
						14	—	$1\frac{1}{4}$	

Before we proceed to the rules of Practice, it will be proper to shew the manner of multiplying and dividing numbers of several denominations, by a single figure, without Reduction.

Example of MULTIPLICATION.

$$\begin{array}{r}
 \text{l. s. d.} \\
 \text{Multiply } 5 \quad 8 \quad 4 \text{ by } 8. \\
 \hline
 \text{facit } 43 \quad 6 \quad 8
 \end{array}$$

Here, first the 4 d. multiply'd by 8, gives 32 d. equal to 2 s. 8 d. therefore setting down 8 in the place of pence, we carry 2 to the produce of the shillings: Again, 8 times 8 s. being 64 s. and the 2 carry'd making it 66 s. or 3 l. 6 s. the 6 is set down in the place of shillings, and the 3 carry'd on to the produce of the pounds. Lastly, 8 times 5 l. being 40 l. and the 3 carry'd making 43 l. the whole product appears to be 43 l. 6 s. 8 d.

Example of DIVISION.

$$\begin{array}{r}
 \text{l. s. d.} \\
 \text{Divide } 43 \quad 6 \quad 8 \text{ by } 8 \\
 \hline
 8) 43 \quad 6 \quad 8(\\
 \hline
 \text{facit } 5 \quad 8 \quad 4
 \end{array}$$

In dividing the above sum, we find, first, that 8 will go 5 times in 43 l. and that 3 will remain; therefore setting down 5 in the place of pounds, we multiply the said remainder by 20, and, taking in the 6 odd shillings, the produce is 66 s. in which again, the divisor going 8 times, and 2 remaining, we set down 8 in the place of shillings, and multiply the 2 remaining by 12, which, with the 8 odd pence.

ence taken in, produces 32 *d.* in which, lastly, the
divisor 8 going exactly 4 times, 4 is set in the
place of pence, and the whole Quotient will be
found to be 5 *l.* 8 *s.* 4 *d.*

Rules of PRACTICE, with Examples.

1. When the price is such a number of pence as
make an even part of a shilling, divide the quan-
tity of goods by such part, and the quotient will
be the answer in shillings.

E X A M P L E.

Suppose you would know the price of 5296 ounces
of Gum, at 6 *d.* per ounce ; then

$$\begin{array}{r} \text{oz.} \\ 6 \text{ d. being } \frac{1}{2} \} 5296 \\ \text{of a Shilling. } \} \hline 2648 \text{ shillings is the answer.} \end{array}$$

For were the price 12 *d.* or 1 *s.* the number of
ounces would be the number of shillings they
would cost : therefore, the price being 6 *d.* the $\frac{1}{2}$
of a shilling, the answer must be but $\frac{1}{2}$ so many
shillings. So at 1 *d.* 1 *d.* $\frac{1}{2}$, 2 *d.* 3 *d.* or 4 *d.* 'tis but
the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{5}$ parts of the quantity of
goods, and the quotient (as aforesaid) will be the
answer in shillings, which, divided by 20, gives
the pounds.

Note, If any thing remains in dividing by the
said parts, (pence being the next denomination) it
must be multiply'd by 12, the pence in 1 shilling ;
and the product divided again by the said part ; and
if any thing remains there, it must be multiply'd
by 4, and still divided by the said part, to produce
farthings.

C 5

E X A M P L E.

EXAMPLE.

	oz.	d.
	5243	at $1\frac{1}{2}$ per Ounce.
d.	<u>65</u>	$5\ 4\frac{1}{2}$
$1\frac{1}{2}\frac{1}{8}$		

$32\text{ l. } 15\text{ s. } 4\text{ d. } \frac{1}{2}\text{ facit.}$

2. If the pence of the price are not an even part of a shilling, it will require more than one division to find the cost of the goods. Thus at 5 d. or any other number of pence between 6 and 12.

EXAMPLES.

lb. - d.	lb. d.
527 at 5 per lb.	436 at 9 per lb.
d. <u>4</u> $\frac{1}{3}$ 175 8	d. <u>6</u> $\frac{1}{2}$ 218
1 $\frac{1}{4}$ 43 11	3 $\frac{1}{2}$ 109
<u>21</u> 9 7	<u>32</u> 7
10 l. 19 s. 7 d. facit.	16 l. 7 s. facit.

In the first of these, 5 d. is taken, thus : 4 d. is the $\frac{1}{3}$ of a shilling, or the top line, and 1 d. the $\frac{1}{4}$ of 4 pence.

In the second, the $\frac{1}{2}$ of a shilling is first taken for 6 d. and then the $\frac{1}{4}$ of that 6 d. for 3 d.

3. If the price is both pence and farthings ; for the pence, as before ; and for the farthings, take the even parts of a penny : thus.

EXAMPLE.

EXAMPLE.

oz. d.
532 at $1 \frac{1}{2}$ per Ounce.

Penny, $1 \frac{1}{2}$ 44 4
Farthing, $\frac{1}{4}$ 11 1

5|5 5

2 l. 15 s. 5 d. facit.

4. When the price is more than 1 s. and less than 2 s. leave the top line for the shilling, and take your parts, as before, for the remaining pence.

EXAMPLE.

lb. d.
5437 at 19 per lb.
d. 6 $\frac{1}{2}$ 2718 6
 $1 \frac{1}{2} \frac{1}{4}$ 679 7 $\frac{1}{2}$

883|5 $1 \frac{1}{2}$

441 l. 15 s. 1 d. $\frac{1}{2}$ facit.

5. If the price is two or more shillings, with pence, &c. either take the even parts of a pound from your top line for the shillings, which will give you the price in pounds; or multiply your top line by the number of shillings, which will give the answer in shillings, still working for the pence as before. See both ways in the example.

EXAMPLE.

oz.	s.	d.	oz.	s.	d.
5263	at 2	6 per Ounce.	5263	at 2	6 per Ounce
s.	d.				
2	6 $\frac{1}{8}$	657 17 6			
			d.		
			6 $\frac{1}{2}$	10526	
				2631	6
				1315	7 6
					657 l. 17 s. 6 d. facit.

By the first way, it is thus done: Had the price been 1 l. or 20 s. 'tis plain it would have come to as many pounds as there were ounces; therefore at 2 s. 6 d. which is the $\frac{1}{8}$ of a pound, it must come to the $\frac{1}{8}$ of so much money.

The other is done by supposing the top line at 1 s. then for 2 s. it must be twice so much; and for the 6 d. half the top line, or the value of a shilling, must be added. See both ways in another

EXAMPLE.

lb.	s.	d.	lb.	s.	d.
5276	at 19	9 per lb.	5276	at 19	9 per lb.
s.					
10 $\frac{1}{2}$	2638			19	
5 $\frac{1}{2}$	1319				
4 $\frac{1}{2}$	1055	4	d.	47484	
6 d. $\frac{1}{2}$	131	18	6 $\frac{1}{2}$	5276	
3 $\frac{1}{2}$	65	19	3 $\frac{1}{2}$	2638	
				1319	
				10420	1
					5210 1 s. facit.
					6. When

6. When the price is any thing between one and two pounds, leave the top line for the pound, and take your parts for the remainder.

EXAMPLE.

	lb.	s.	d.
s. d.	5384	at 22	6 per lb.
2 6 $\frac{1}{8}$	673		
	6057 l. facit.		

7. When the price is two or more pounds, multiply the top line by the number of pounds; and for the remainder, take parts, as before.

EXAMPLE.

	lb.	l.	s.	d.
	2364	at 4	18	10 $\frac{1}{2}$ per lb.
	4			
s.	9456			
10 $\frac{1}{2}$	1182			
5 $\frac{1}{2}$	591			
2 $\frac{1}{2}$	263	8		
1 $\frac{1}{2}$	118	4		
6d. $\frac{1}{2}$	59	2		
3 $\frac{1}{2}$	29	11		
1 $\frac{1}{2}$	14	15	6	
	11687 l. 00s. 6d. facit.			

8. When with the given quantity there are odd parts, such parts must be taken from the given price; as in the following examples.

EXAMPLES.

[50]

E X A M P L E S.

C. grs. l. s.
365 2 at 5 5 per C.
5

s. 1825
5 1/2 91 5
2 grs. 1/2 2 12 6

1918 l. 17 s. 6 d. facit:

C. grs. lb. l. s. d.
7365 3 14 at 8 19 6 per C.
8

s. 58920
10 1/2 3682 10
5 1/2 1841 5
4 1/2 1473 0
6 d. 1/8 184 2 6
2 grs. 1/2 4 9 9
1 1/2 2 4 10 1/2
14 lb 1/2 1 2 5 1/2

66108 l. 14 s. 6 d. 1/2 facit.

As in the first of these, for the 2 grs. 1/2 the price of the C. is taken; so in the second, not only 1/2 the price of the C. is taken for 2 grs, but also 1/2 the price of 2 grs. for 1 gr, and 1/2 that again for 14 lb. See more examples.

E X A M P L E S.

[51]

EXAMPLES.

Tons. C. qrs. lb. l. s. d.
573 13 2 18 at 9 12 10 per Ton.

9

		5157	
10s.	$\frac{1}{2}$	286	10
2	$\frac{1}{2}$	57	06
0 d.	$\frac{1}{2}$	23	17 6
10 C.	$\frac{1}{2}$	4	16 5
2	$\frac{1}{2}$	19	3 $\frac{1}{4}$
1	$\frac{1}{2}$	09	7 $\frac{1}{4}$
2 qrs.	$\frac{1}{2}$	04	9 $\frac{1}{4}$
14 lb.	$\frac{1}{2}$	01	2 $\frac{1}{4}$
2	$\frac{1}{2}$		2
2	$\frac{1}{2}$		2

5531 l. 5s. 1d $\frac{1}{4}$

lb. oz. drams. l. s. d.
527 13 15 at 2 8 7 $\frac{1}{2}$ per lb.

2

		1054	
5s.	$\frac{1}{4}$	135	15
2 10	$\frac{1}{4}$	52	14
1	$\frac{1}{4}$	26	07
6 d.	$\frac{1}{4}$	13	03 06
1 $\frac{1}{2}$	$\frac{1}{4}$	3	05 10 $\frac{1}{2}$
8 oz.	$\frac{1}{4}$	1	04 03 $\frac{1}{4}$
4	$\frac{1}{4}$	12	01 $\frac{1}{4}$
1	$\frac{1}{4}$	03	00 $\frac{1}{4}$
drs.	$\frac{1}{4}$	01	06
4	$\frac{1}{4}$		09
2	$\frac{1}{4}$		04 $\frac{1}{4}$
1	$\frac{1}{4}$		02 $\frac{1}{4}$

1283 l. 7s. 8d. *fact.*

The

[52]

The last Sum another way.

lb. oz. drams. l. s. d.
 527 13 15 at 2 8 7 $\frac{1}{2}$ per lb.
 48

4216
 2108
 6d. $\frac{1}{2}$ — 263 6
 1 $\frac{1}{2}$ $\frac{1}{4}$ — 65 10 $\frac{1}{2}$
 8 oz. $\frac{1}{2}$ — 24 3 $\frac{1}{4}$
 4 — $\frac{1}{2}$ — 12 1 $\frac{1}{4}$
 1 — $\frac{1}{4}$ — 3 0 $\frac{1}{4}$
 8 dr. $\frac{1}{2}$ — 1 6 —
 4 — $\frac{1}{2}$ — 9 —
 2 — $\frac{1}{2}$ — 4 $\frac{1}{2}$
 1 — $\frac{1}{2}$ — 2 $\frac{1}{4}$
 2566 | 7 8

1283 l. 7 s. 8 d. factt.

Another EXAMPLE.

lb. Troy. oz. dwts. grs. l. s. d.
 763 11 12 13 at 23 16 4 per lb.
 23

2208
 1472
 10 s. — 368
 5 — 184
 1 — 36 16
 4 d. — 12 5 4
 6 oz. — 11 18 2
 4 — 7 18 9 $\frac{1}{2}$
 1 — 1 19 8 $\frac{1}{2}$
 16 dwts. — 19 16
 2 — 3 11
 12 grs. — 11
 1 —

17552 l. 2 s. 9 d. $\frac{1}{2}$ factt.

Not

Note. The foregoing examples, and all other questions in *Practice*, may be perform'd by the *Rule of Three*, *Practice* being only a compendious way of calculating such propositions in proportion, as have unite for their first number: But to shew the brevity of this way of working, as well as to prove the truth thereof; see the last example done by the *Rule of Three*, as follows.

By the *Rule of Three* thus :

lb. l. s. d. lb. oz. dwts. grs.
1—cost—23 16 4 what will 736 11 12 13?

2	20		12
2	476		8843
20	12		20
40	5716		176872
24			24
60			707491
0			353745
60			4244941
			5716
			25465646
			4244941
			29714587
			21224705
			12
576	0	2426408275	6
		4212514	10
		1221	
		20	
		17552	2 10 1

Cost 17552 l. 2 s. 10 d. 1.

Three Farthings more
than by *Practice* lost
here, by the *Fractions*
remaining.

1448
2062
827
2515
3116
4
576
0
346
4
370

PROOF

PROOF.

Sums in this rule may be prov'd either by taking the parts two ways, as the last example but one ; or by the *Rule of Three*, as the last.

ABBREVIATIONS.

As this whole rule is indeed nothing else, it may be thought strange to mention short ways here, in a particular section; but as there are degrees of comparison, so even short ways may be more shorten'd.

1. The value of any quantity (the given price of one which is an even number of shillings) may be most compendiously found, by multiplying the said quantity by $\frac{1}{2}$ their number, doubling the unites of the first product for shillings, the rest being pounds.

EXAMPLE.

5276 Ells of Holland, at 8 s. per Ell.

4

21104 Answer.

2. If the price is an odd number of shillings, it is but doing as before, and adding $\frac{1}{2}$ of the top line to the product.

EXAMPLE.

Thirds: s.
5278 at 15 per third.

7

3684112
263118

15: 28

39581: 10: Answer.

3: Were

Were there pence in the price, their parts might also easily be taken in the same method.

EXAMPLE.

lb. s. d.
5384 at 7 7½ per lb.
3

1 s.	28	1615	4
6 d.	12	269	4
1 1	1	134	12
		33	13

2052 l. 13 s. facit.

Another short and useful way, when the quantity is small, is to multiply the price by it; thus our quantity is express'd by one figure, 'tis done one line.

EXAMPLE.

5 Yards of Velvet, at 1 13 6 per Yrd.
5

8 l. 7 s. 6 d. facit.

So any quantity that can be expressed by 2 figures, and some of 3, may be easily and speedily done thus: Find out what two figures multiply'd together, will produce the given quantity, (as sup- 32 yards or pounds, &c. the given quantity, which is produced by the multiplication of 8 by 4) multiply the price by one of the said figures; the product of that, multiply'd again by the other, will give the answer.

EXAMPLES.

EXAMPLES.

Yards. s. d.
32 at 15 8 $\frac{1}{2}$ per Yard.
8

The value of 8 yards 6 5 8—
which multiply'd by 4

gives 25 l. 2 s. 8 d. the price of 32 yards
—4 times 8 being

Yards. l. s. d.
144 at 1 5 6 per Yard.
12
15 6 0
12

183 l. 12 s. 0 d. facit.

6. If no two figures will exactly produce the given, take such as will give the nearest product them, which will generally be within one or two more or less; the value of which must accordingly be added or subtracted.

EXAMPLE.

lb. l. s. d.
55 at 1 10 9 per lb.
9

The price of 9 lb. 13 16 9
which multiply by 6

gives the Price of 54 lb. 83 00 6
to which the price of 1 lb added 1 10 9

makes 84 11 3 the value of

So had it been multiply'd by 7 and 8, which would
 ve given the value of 56 lb too much by 1 lb, the
 ce of 1 lb must have been substracted.

7. If the quantity has odd parts, it will be easy
 ing the value of such parts from the given price;
 in the following

E X A M P L E.

Rds. *l. s. d.*
 43 $\frac{1}{2}$ at 2 12 7 per Yard.

7

 18 8 1
 6

qrs. *l. s. d.*
 2 $\frac{1}{2}$ 2 12 7
 1 6 3 $\frac{1}{2}$

 114 l. 7 s. 4 $\frac{1}{2}$ facit.

T R E T T a n d T A R E.

T A R E and **T R E T T** are allowances made to
 Merchants in buying their goods.

Tare, of what they can agree for *per* the whole,
 chest, &c. or *per* C. for the weight of the bag,
 x, chest, &c. which contains the commodity.

Trett, for the waste, motes, or dust; and is al-
 ways 4 lb. *per* 104 lb.

There is also another allowance sometimes given
 2 lb. for every 3 C. for the turn of the scale, call'd
 off, or elough.

Note, The whole weight, before any allowances
 are made, is call'd *gross*; when part is deducted,
 the remainder is call'd *nettle*; but when all are
 taken from it, what's left, is call'd *neat*.

R U L E S

RULES, with EXAMPLES.

1. The *Tare* of any quantity of goods may easily be found, if at so much in the whole, only by subtracting the said allowance from the gross weight; if at so much *per chest*, &c. by multiplying the pounds tare by the number of chests, &c. and subtracting as before; and if at so much *per C.* by taking such part, or parts, of the gross weight, as the allowance is of a C.

EXAMPLES.

First sort. Suppose 15 C. 2 qrs. 13 lb. tare was allowed on 456 C. 1 qr. 19 lb. of tobacco, what would be the neat weight?

	C.	qrs.	lb.	
	From	456	1	19
				Gross.
	Subtract	15	2	13
				Tare.

Remainder 440 3 6 Neat.

Second sort. What's the neat weight of 3 frails of raisins, each weighing 3 C. 2 qrs. 10 lb. gross; tare at 20 lb. per frail?

C.	qrs.	lb.		lb.
3	2	10		20
			3 Frails.	3 Frails.
10	3	2	Gross.	60 lb. or 2 qrs. 4 lb.
	2	4	Tare.	

Answer, 10 0 26 Neat.

C.	qrs.	lb.		lb.
246	3	12	Gross.	Tare 14 per C.
14	$\frac{1}{8}$	30	3	12 Tare.

Answer, 216 0 0 Neat.

C. grs. lb. lb.
Again, 264 1 19 Gross. Tare 16 per C.

lb. 16 7 52 0 6 1/2 Tare.

Answer, 212 1 12 1/2 Neat.

Note, 14 and 16 pounds may be call'd the standards of Tare; for from them may any other number of pounds, more or less, be taken, as in the following

E X A M P L E S.

C. grs. lb. lb.
573 2 13 Gross. Tare 17 per C.

lb. 14 1/4 71 2 22 1/2
2 1/4 10 0 27 —
1 1/4 5 0 13 1/2

87 0 7 Tare.

Answer, 486 2 6 Neat.

C. grs. lb. lb.
548 3 11 Gross. Tare 10 per C.

lb. 16 7 78 1 17 1/2

8 1/4 39 0 23 1/2
2 1/4 9 3 5 1/2

49 0 0 1/4 Tare.

Answer, 499 3 10 1/4 Neat.

Note, 'Tis first found what tare 16 lb. would produce; because 8 lb. being the 1/2 part of a C. would be a troublesome divisor.

2. *Trett* being always 4 lb. per 104 lb. the constant method of finding it, is by taking the $\frac{1}{26}$ part of the line it is to be deducted from, 4 times 26 being 104. But it being difficult to divide by 26 in ordinary line, it will be more safe and easy to do the work aside, as in the first example.

EXAMPLES.

	C.	qrs.	lb.	lb.	
	428	1	19	Gross.	<i>Trett</i> 4 per 104
lb.					
4 $\frac{1}{2}$	16	1	25 $\frac{1}{2}$	<i>Trett.</i>	26)428(16
<i>Facit.</i>	411	3	21 $\frac{1}{2}$	<i>Neat.</i>	168
					12
					4
					26)49(18
					23
					28
					193
					47
					26)663(10
					141
					13
					4
					26)51(19
					10

C.	grs.	lb.		lb.
836	2	17	Gross.	Tare 22 per C.
			Tret, 4 lb. per 104 lb.	

119	2	2 $\frac{1}{4}$
29	3	14 $\frac{1}{4}$
14	3	21 $\frac{1}{4}$

164	1	10 Tare.
-----	---	----------

672	1	7 Suttle.
-----	---	-----------

25	3	12 Trett.
----	---	-----------

646	1	23 Near.
-----	---	----------

3. *Clough* (which is 2 lb for every 3 C.) is found, first dividing the hundreds of the line it is to be taken from by 3, which brings them into 3 C's; then 2 lb. being to be allow'd for every 3 C. as many C's as it produces, so many 2 lb's it will allow; then dividing 56, (the double pounds in a hundred) the quotient will be hundreds, and the remainder double pounds; to which adding, what may be allow'd for the odd hundreds, quarters, and pounds of the given weight, it makes the whole *Clough*; which subtracted from the said given weight, leaves the *Neat*.

D

EXAMPLES.

EXAMPLES.

C.	grs.	lb.		lb.	C.
5647	3	13	Gross.	Clough 2 for 3	
33	2	13	$\frac{1}{2}$ Clough.	3)5647(1	
				C.	grs. lb.
5614	0	27	$\frac{3}{4}$ Neat.	56)1882(33	2
				168	
				202	33 2
				168	
				14)34(2	
				28	
				6 double Pounds	

Note, The remainder 34 (which are double pounds) is divided by 14, (the double pounds in a gr.) bring the said remainder into quarters of an hundred.

Note also, The 1 lb. $\frac{1}{4}$ is allow'd for the odd hundred, 3 grs. and 13 lb.

EXAMPLE 2.

What's the neat weight of 3 hogheads of tobacco weighing *viz.*

Num

Number C. grs. lb.

—	5	3	18
—	4	2	11
—	5	1	19

15 3 20 Gross.

{ Tare 7 lb. per C.
Trett 4. lb. per 104 lb.
Clough 2 lb. for 3 C.

14 $\frac{1}{8}$ 1 3 27 $\frac{1}{2}$

7 $\frac{1}{2}$ 3 27 $\frac{1}{2}$ Tare.

14 3 20 $\frac{1}{2}$ Suttle.

2 $\frac{1}{8}$ 2 8 $\frac{1}{4}$ Trett.

14 1 12 $\frac{1}{4}$ Suttle.

9 $\frac{1}{2}$ Clough.

C.
3)14(4

Net, 14 1 2 $\frac{1}{4}$ Neat.

2
lb.

Note, The 4 three hundreds allow — 8
And the odd 2 C. 1 gr. 14 lb. $\frac{1}{2}$ allow — 1 $\frac{1}{2}$
Total Clough 9 $\frac{1}{2}$





CHAP. V.

Of INTEREST.

INTEREST is either { Simple, { which arises only from the Principal.
or { Compound, { which arises both from Principal and Interest.

SIMPLE INTEREST.

SIMPLE INTEREST being one of the branches of Proportion, or the Rule of Three, is worked as follows.

1. The Interest of any Sum for one year is found by this plain proportion, *viz.*

As 100 l.
To the rate of Interest :
So is the given Principal,
To the Interest required.

EXAMPLE.

What's the Interest of 429 l. for a year, at 5 per. Cent. per Annum?

l. l. l.
If 100 — 6 — 429
6

25 | 74
20

14 | 80
12

9 | 60
4

2 | 40

Note, The cutting off two figures to the right and, divides such lines by 100. See Abbreviations Division.

2. To find the Interest of any sum for 1 or more years, multiply the year's amount by the number years required.

EXAMPLE.

What interest will the foregoing principal produce in 4 years, at the same rate of Interest?

l. s. d.
25 14 9 $\frac{1}{2}$ the Interest for one Year.
4

Facit 102 19 2

3. To find interest for months, divide the year's amount by the even parts they make of a year.

D 3

EXAMPLE.

[67]

Then, *Weeks. l. s. d. Weeks.*
 If 52 — 19 6 2 $\frac{1}{4}$ — 3

20

386

12

4634

4

18537

3 4

52) 55611 (1069 (1

52

12) 167 (3

361

312

312

1 l. 2 s. 3 d. $\frac{1}{4}$ factv.

491

469

23

Qn. 2. What interest is now due on a bond of 10 l. 10 s. commencing interest at 5 l. 10 s. per cent per Annum, July the 10th, 1727; this being arch the 25th, 1729?

l. l. s. l. s.
 100 — 5 10 — 420 10

5

s. 2102 10

10 $\frac{1}{2}$ 210 5

23 | 12 15

20

2 | 55

12

6 | 60

4

2 | 40

July — 21

August — 31

September — 30

October — 31

November — 30

December — 31

January — 31

February — 28

March — 25

258

3 l. 2 s. 6 d. $\frac{1}{2}$
 for a Year.

D 4 Then,

Then, If *Days. l. s. d.* *Days.*
 $365 - 23 \ 2 \ 6\frac{1}{2} = 258$

$$\begin{array}{r}
 20 \\
 \hline
 462 \\
 12 \\
 \hline
 5550 \\
 4 \\
 \hline
 22202 \\
 258 \\
 \hline
 177616 \\
 111010 \\
 44404 \\
 \hline
 365) 5728116(15693(1 \\
 \underline{365} \qquad \qquad \qquad 12)3923(11 \\
 2078 \qquad \qquad \qquad \underline{210)32|6} \\
 1825 \qquad \qquad \qquad \hline
 2531 \qquad \qquad \qquad 16 \text{ L. } 6 \text{ s. } 11 \text{ d } \frac{1}{2} \text{ for } 258 \text{ days} \\
 \underline{2190} \qquad \qquad \qquad 23 \text{ l. } 2 \text{ s. } 6 \text{ d } \frac{1}{2} \text{ for a year.} \\
 3411 \qquad \qquad \qquad \hline
 \underline{3285} \qquad \qquad \qquad 39 \text{ l. } 9 \text{ s. } 5 \text{ d } \frac{1}{2} \text{ total Interest} \\
 1266 \\
 \underline{1095} \\
 171
 \end{array}$$

Another way to cast up Interest, when the principal has odd money in it, and the time, weeks or days.

R U L E.

Bring your principal into its lowest denomination, and multiply it by the number of days; and,

If at 5 *l. per Cent.* divide the product by 7300, the quotient gives the answer in the same denomination the principal was reduced into. *Note,* The number 7300 is found by multiplying 365 by 100, and dividing by 5.

EXAMPLES.

What's the Interest of 432 *l.* 12 *s.* 6 *d.* for 8 days, at 5 *l. per Cent. per Annum*?

l. s. d.
432 12 6
20

8652
12

103830
8 12

73|00)8306|40 (113
73

9 *s.* 5 *d.* $\frac{1}{4}$ *facit.*

100
73

276
219

5740
4

73|00)229|60 (3
219

10

If you would cast up Interest at 6 *l. per Cent.* by this rule you must add $\frac{1}{4}$ of the quotient to the interest produced at 5 *l. per Cent.* and so at other rates,
D 5 ic

it is but adding or subtracting such and such parts of the quotient to or from what it produces at 5. For instance, suppose the last question done the common way, for an

EXAMPLE.

l. s.
420 10 Principal.
10

8410
623 number of days.

25230
16820
50450

73|00) 5239430 (717 8 4
511

10 10 71 9 4
129
74 78|9 6

564 l. 39 9 6
511

Answer, 1 farthing more than by the other way.

5330
12

73|00) 63960 (8
584

5560
4

73|00) 22240 (3
219

340

Note, The Rate here being 5 l. 10 s. and 10 s. being $\frac{1}{10}$ of 5 l. there is therefore added $\frac{1}{10}$ of the quotient, or the amount, at 5 l. per Cent.

More Examples in Simple Interest.

At 6 l. per Cent. what will 489 l. come to in a year?

Answer, 35 l. 6 s. 9 d $\frac{1}{4}$.

What will the same sum, at the same rate, amount to in 8 years?

Answer, 282 l. 14 s. 4 d.

What's the Interest of 7030 l. for 5 months, at 5 l. per Cent. per Annum?

Answer, 146 l. 5 s.

What's a week's Interest of 1000 l. at 7 l. per Cent.?

Answer, 1 l. 6 s. 11 d.

What will the Interest of 5439 l. 10 s. 8 d. come to in 20 days, at 5 l. per Cent.?

Answer, 14 l. 17 s. 6 d.

What Interest is now due on a Navy-bill, commencing Interest at 6 l. per Cent. per Annum, June the 17th 1726, this being January the 10th 1728, the principal being 836 l.

Answer, 8 l. 7 s. 10 d.

Compound Interest.

COMPOUND INTEREST, as was before said, arises both from the principal and interest, (i. e.) when interest on money becoming due, and not paid, the same rate is allow'd on the unpaid interest, as was before on the principal.

Therefore, to work sums in this rule, after having found the first year's interest, as before, add it to the principal, and find the interest of the sum; and so continue to add every year's produce, still accounting the sum a new principal.

E X A M P L E.

What will be the amount of 400 *l.* forborn 3 years and $\frac{1}{2}$ at 6 *l. per Cent. per Annum*, compound interest?

$\begin{array}{r} \text{l.} \quad \text{l.} \quad \text{l.} \\ \text{If } 100 \text{ --- } 6 \text{ --- } 400 \end{array}$
Principal 1st year.

$\begin{array}{r} \text{l. } 24 | 00 \\ \hline \end{array}$

$\begin{array}{r} \text{l.} \quad \text{l.} \quad \text{l.} \\ \text{If } 100 \text{ --- } 6 \text{ --- } 424 \end{array}$
Principal 2d. year.

$\begin{array}{r} \text{l. } 25 | 44 \\ \hline 20 \end{array}$

$\begin{array}{r} \text{s. } 8 | 80 \\ \hline 12 \end{array}$

$\begin{array}{r} \text{d. } 9 | 60 \\ \hline 4 \end{array}$

$\begin{array}{r} \hline 2 | 40 \end{array}$

Principal 10

424
25 8 9 $\frac{1}{2}$

l. 100 — l. 6 — 449 8 9 $\frac{1}{2}$ Princip. 3d year.
6

l. 26 | 96 12 9 —
20

s. 19 | 32
12

d. 3 | 93
4

3 | 72

l. s. d.
449 8 9 $\frac{1}{2}$
26 19 3 $\frac{1}{2}$

l. 100 — l. 6 — 476 8 1 $\frac{1}{2}$ Princip. 4th year.
6

l. s. d. l. 28 | 58 8 7 $\frac{1}{2}$
28 11 8 20

M. 6 $\frac{1}{2}$ 14 5 10 s. 11 | 68
476 8 1 $\frac{1}{2}$ 12

490 13 11 $\frac{1}{2}$ d. 8 | 23
Total amount. 4

94

Or, $\frac{1}{2}$

Or, more plain :

	<i>l.</i>	<i>s.</i>	<i>d.</i>
<i>Interest the 1st Year</i>	24	0	0—
<i>2^d ———</i>	25	8	9 $\frac{1}{4}$
<i>3^d ———</i>	26	19	3 $\frac{1}{4}$
<i>— 1st Year —</i>	14	5	10
<i>Total Interest</i>	90	13	11 $\frac{1}{4}$
<i>Principal</i>	400	0	0
<i>Total amount</i>	490	13	11 $\frac{1}{4}$

More EXAMPLES.

What will 1000*l.* amount to in 3 years, at 6*per Cent. per Annum*, Compound Interest?

Answer, 1191*l.* 0*s.* 3*d.* $\frac{1}{4}$.

At 5*l.* 10*s.* *per Cent. per Annum*, Compound Interest, what will 560*l.* 10*s.* 9*d.* amount to, forborn 5 years and an half?

Answer, 752*l.* 14*s.* 10*d.* $\frac{1}{2}$.



THE ART AND MYSTERY OF ARITHMETIC

CH A P. VI.

Of REBATE, or DISCOUNT.

REBATE, or DISCOUNT, is the abating so much money on a debt paid before 'tis due, as might be gain'd again by the money receiv'd, if put out to interest at the same rate, and for the same time. So 100 *l.* present money, would discharge a debt of 106 *l.* due at a year to come; rebate being made at 6 *l.* per Cent. because 100 *l.* put out to interest for a year, at the said rate, would regain the 6 *l.*

To work Sums in this RULE.

First find the interest of 100 *l.* for the time; then by a *Rule of Three* stating, of which the first number must be 100 *l.* with the interest found; the second number, the interest alone (or 100 *l.* alone, if you would find the present money); and the third, the debt, or sum propounded, you will find the answer.

EXAMPLES.

EXAMPLES.

EX. 1. What's the rebate of 420 l. for a year,
6 l. per Cent. per Annum?

$$\begin{array}{r}
 \text{l.} \quad \text{l.} \quad \text{l.} \\
 \text{If } 106 \text{ --- } 6 \text{ --- } 420 \\
 \quad \quad \quad 6 \\
 \quad \quad \quad \text{---} \text{l.} \\
 \quad \quad 106) 2520 (23 \\
 \quad \quad \quad 212 \\
 \quad \quad \quad \text{---} \\
 \quad \quad \quad 400 \\
 \quad \quad \quad 318 \\
 \quad \quad \quad \text{---} \\
 \quad \quad \quad 82 \\
 \quad \quad \quad 20 \\
 \quad \quad \quad \text{---} \text{s.}
 \end{array}$$

Ans. 23 l. 15 s. 5 d $\frac{1}{2}$. 106) 1640 (15

$$\begin{array}{r}
 106 \\
 \text{---} \\
 \cdot 580 \\
 530 \\
 \text{---} \\
 \cdot 50 \\
 12 \\
 \text{---} \text{d.} \\
 106) 600 (5 \\
 530 \\
 \text{---} \\
 \cdot 70 \\
 4 \\
 \text{---} \\
 106) 280 (2 \\
 212 \\
 \text{---} \\
 \cdot 68
 \end{array}$$

year,

Ex. 2. What present money will discharge a debt
732 l. due at 3 months, rebate being made at 6 l.
per Cent. per Annum?

l. 1. 6 Interest for a Year of 100
Months—
3 $\frac{1}{4}$ 1 10

Then, if 101 10 fall to 100, what will 732
2 2

203

203)146400(721
1421

410
406

240
203

37
20

Ans. 721 l. 3 s. 7 d $\frac{1}{2}$.

203)740(3
609

131
12

203)1572(7
1421

151
4

203)604(2
406
098

Ex. 1

Ex. 3.

Ex. 3. What rebate must be allow'd on a Bill of Exchange for 85 l. 10 s. due September the 8th, being July the 4th, rebate being made at 5 l. per Cent. per Annum?

		Days.	l.	Days.
July	27	15	5	66
Aug.	31			5
Sept.	8			330
	<u>66</u>			20

365	6600	18	0
	365		
	2950		
	2920		
	30		
	12		
	360		
	4		
365	1440	3	
	1095		
	345		

7) 272 (302

1001

272

302

302

302

302

302

302

302

302

302

302

302

302

302

302

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answer

Ex. 4.
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ade a

Not

Usan
months
ally all
ng the

Then,

When,	l. s. d.	l. s. d.	l. s.
100 18 0	18 0 4	18 0 4	85 10
20	12	20	
2018	216	1710	
12	4	12	
24216	867	10520	
4		4	
96867		82080	
		867	

574560
 492480
 658640
 4
 96867) 71162360 (734(2
 678069
 12 (182(2
 335646
 290601 15 s. 3 d.
 450450
 387468
 62982

Ex. 4. A Bill of Exchange for 250 l. is dated at Amsterdam, June the 14th new stile; at usance is accepted, and payment offer'd the 23th ~~days~~ old stile, what must be then receiv'd, rebate being made at 6 l. per Cent. per Annum?

Note, New Stile is 11 days before Old Stile.

Usance signifies a month; double Usance, two months, &c. Note also, Merchants of London generally allow 3 days beyond the time appointed, calling them 3 days of grace. Therefore the bill being dated

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dated June the 4th N. S. or the 2d, O. S. and accepted at usance, or a month, it becomes due the 2d, O. S. but 3 days of grace being allowed makes it the 6th. So if the money is paid the 1st of June, O. S. rebate must be made for 21 days. Therefore,

Days.		Days.
If 365	— 6 —	21
		6
		126
		20
		— 3.
		365) 2520 (6
		2150
		—
		330
		12
		— d.
Facit 6s. 10d 1/2.		365) 2960 (10
		365
		—
		310
		4
		—
		365) 1240 (3
		1095
		—
		145

A Bill of exchange for 2000 l. is dated the 1st of June 1750 and is payable to the order of the Bank of England the 1st of July 1750. The Bill is drawn on the Bank of England and is payable to the order of the Bank of England. The Bill is drawn on the Bank of England and is payable to the order of the Bank of England.

The Bill is drawn on the Bank of England and is payable to the order of the Bank of England. The Bill is drawn on the Bank of England and is payable to the order of the Bank of England. The Bill is drawn on the Bank of England and is payable to the order of the Bank of England.

[81]

and as
due 7
allow
ne 1st
days

l.	s.	d.	l.	l.
100	6	10	100	250
20				20
1006				5000
12				12
1082				60000
4				4
6331			96331	24000000 (249

192662

473380

385324

880560

866979

13581

20

96331) 271620 (2

192662

78958

12

96331) 947496 (9

866979

80517

4

96331) 322068 (3

288993

33065

Answer, 249 l. 2 s. 9 d.

far.

More

MORE EXAMPLES

What's the rebate of 100 $\text{\textsterling}$. for $\frac{1}{2}$ a year, at 6 per Cent. per Annum?

Answer, 2 $\text{\textsterling}$. 18 s . 3 d .

What's the rebate of 538 $\text{\textsterling}$. 10 s . for 5 months, at 5 $\text{\textsterling}$. per Cent. per Annum?

Answer, 10 $\text{\textsterling}$. 19 s . 9 d . $\frac{1}{2}$.

What present money is a debt of 629 $\text{\textsterling}$. due at 10 weeks worth, rebate being made after the rate of 6 $\text{\textsterling}$. 10 s . per Cent. per Annum?

Answer, 626 $\text{\textsterling}$. 13 s .

What present money will discharge 10000 $\text{\textsterling}$. due at 5 days, rebate being made at 8 $\text{\textsterling}$. per Cent. per Annum?

Answer, 9989 $\text{\textsterling}$. 1 s . 5 d . $\frac{1}{2}$.



C H A P. VII.

EQUATION of PAYMENTS.

THE Design of this rule is, when several sums are due at several times, to find a mean time for paying the whole debt.

The common way to do which, is,

Multiply each sum by its respective time, and add the products together, dividing their total by the whole debt; the Quotient is accounted the mean time of paying the whole debt.

E X A M P L E.

A. owes to B. 40 l. to pay at 3 months, 60 l. at 5 months, and 100 l. at 10 months; at what time may the whole debt be paid together, without injury to either?

l.	l.	l.	l.
40	40	60	100
60	3	5	10
100			
	120	300	1000
200			300
			120

$$2|00) 14|200$$

Answer, 7 months.

Note,
†

Ans. The times of payment, if not so given, must be reduced into one name.

But those who are more exact, take the pains to work these sums by the following rule,

First they find the present worth of every particular sum; then find in what time to come the sum of these present worths will be encreas'd to the total of all the particular sums payable at several times to come, and that is the true equated time for payment of the whole.



CHA

C H A P. VIII.

OF FELLOWSHIP, or COMPANY.

FELLOWSHIP, or COMPANY, is when two or more, join their stocks, and trade together, dividing their gain or loss proportionably.

FELLOWSHIP is either with, or without time. Questions without time, are work'd by this proportion :

As the whole stock,
To the whole gain, or loss:
So is each man's particular stock,
To his particular share.

E X A M P L E S.

A. B. and C. make a joint-stock; A. puts in 460 *l.* 10 *l.* and C. 480 *l.* they gain 340 *l.* what part of belongs to each?

	<i>l.</i>
A. _____	460
B. _____	510
C. _____	480
	—

Total stock 1450

1st. Then, If	^{l.} 145	^{l.} 10	^{l.} 340	^{l.} 46	10
				340	
2d. If	^{l.} 145	^{l.} 10	^{l.} 340	^{l.} 51	10
				340	
			2040		
			153		
				145	15640(10
			145	17340(119	
				145	1140
					1015
				284	
				145	125
					20
				1390	
				1305	145)2500(
					145
				85	1050
				20	1015
				145	1700(11
				145	35
					12
				250	
				145	145)420(
					290
				105	
				12	130
					4
				145	1260(8
				1160	
				100	
				4	
				145	400(2
				290	
				110	

's Sh
's Sh
's Sh
lue e
-
e wh
gain

And

[87]

And, if $\begin{array}{r} l. \\ 145 | 0 \end{array} \begin{array}{r} l. \\ 340 \end{array} \begin{array}{r} l. \\ 48 | 0 \end{array}$

$\begin{array}{r} 340 \\ \hline \end{array}$

1910

144

$\begin{array}{r} 145) 16320 (112 \\ 145 \end{array}$

182

145

370

290

80

20

$\begin{array}{r} 145) 1600 (11 \\ 145 \end{array}$

150

145

5

12

60

4

$\begin{array}{r} 145) 240 (1 \\ 145 \end{array}$

95

	l.	s.	d.	Rem.
's Share is	107	17	2	$\frac{1}{4}$ — 85
's Share is	119	11	8	$\frac{1}{4}$ — 110
's Share is	112	11	0	$\frac{1}{4}$ — 95
Value of the Remainder				$\frac{1}{4}$ —
				145)290(2
the whole				290
gain	340	00	0	—
				00

And thus are these Sums prov'd.

More *EXAMPLES.*

Three persons make a joint-stock; *A* puts 565 *l.* *B.* 278 *l.* and *C.* 629. they gain 1000 *l.* what each of their shares?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	383	16	7 $\frac{1}{2}$	384
{ <i>B.</i>	188	17	2	512
{ <i>C.</i>	427	6	2 $\frac{1}{4}$	576

D. *E.* and *F.* trade together; *D.* puts in 220 *l.* *E.* 529 *l.* and *F.* 344 *l.* 10 *s.* their whole gain amounts to 520 *l.* what is that to each?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	
<i>Answer,</i> { <i>D.</i>	108	7	7 $\frac{3}{4}$	271
{ <i>E.</i>	249	5	7	844
{ <i>F.</i>	162	6	9	1092

A. *B.* and *C.* make a bank of 3256 *l.* whereof *A.* puts in 1026 *l.* *B.* 985 *l.* and *C.* the rest; by misfortune they lose 2000 *l.* what part of it must each bear?

				<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	630	4	5	928
{ <i>B.</i>	605	0	8 $\frac{1}{4}$	1240
{ <i>C.</i>	764	14	10	1088

FELLOWSHIP with TIME.

FELLOWSHIP with TIME is thus work'd: multiply each man's stock by the respective time he puts it in for, and add all the products, the total of which must be your first number through the statings; the gain, or loss, the second (as before)

re); and each man's particular stock, multiply'd by its time, the third.

Note, All the particular times (if not so given) must be reduced into one name, (*i. e.*) all years, all months, all weeks, or all days, &c.

The reason of multiplying the several stocks by the times they are put in for, will appear from the following considerations. *First*, suppose *A.* and *B.* put into trade each 100 *l.* for 1 month, then certainly their gain, whatever it be, must be equally divided betwixt them: Again, suppose *A.* puts in 100 *l.* and *B.* 200 *l.* both for the same time, 'tis as plain that *B.*'s share of the gain must be twice as much as *A.*'s, because his stock is double: or if each put in 100 *l.* but for different times, *viz.* *A.* for 1 month, and *B.* for 2, then as *B.*'s stock lies twice as long as *A.*'s, so his gain, as before, must be twice as great. *Lastly*, suppose *A.* to put in 100 *l.* for 1 month, and *B.* 200 *l.* for 2 months; then since his stock is not only double, but the time he leaves it double too, his share of the profits must become four times greater than *A.*'s. And so in all other cases.

E X A M P L E.

A. put into company 560 *l.* for 8 months, *B.* 279 *l.* for 10 months, and *C.* 735 *l.* for 6 months; they divid 1000 *l.* What share of it must each have?

<i>l.</i>	<i>l.</i>	<i>l.</i>
560	279	735
8	10	6
<i>A.</i> —	<i>B.</i> —	<i>C.</i> —
4480	2790	4410
		2790
		4480
		11680

E 3

17.

[90]

1/3. Then, 1/ ^{l.} 1168 | 0 — ^{l.} 1000 — ^{l.} 448 | 0
₁₀₀₀

^{l.}
 1168) 448000 (383
 3504

^{l.}
 9760
 9344

^{l.}
 4160
 3504

^{l.}
 656
 20

^{s.}
 1168) 13120 (11
 1168

^{s.}
 1440
 1168

^{s.}
 272
 12

^{d.}
 1168) 3264 (2
 2336

^{d.}
 928
 4

^{d.}
 1168) 3712 (3
 3504
 208

[92]

2d, 7f 1168|0 — ^{l.} 1000 — 279|0
1000

^{l.}
1168)279000(238
2336

4540
3504

10360
9344

1016
20

^{s.}
1168)20320(17
1168

8640
8176

464
12

^{d.}
1168)5568(4
4672

896
4

1168)3584(3
3504
80

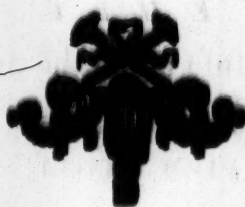
More **EXAMPLES.**

A. B. and C. agreeing to trade together, A. puts in 92 l. for 4 months; B. 329 l. for 7 months; and C. 100 l. for 2 months; by their trade they gain 540 l. What part of it belongs to each?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	183	14	8	2304
{ <i>B.</i>	199	19	5	1332
{ <i>C.</i>	156	5	10 $\frac{1}{4}$	2583

Three persons make a joint-stock; *A.* puts in 400 pieces of holland, each containing 56 yards, at 5 s. *d.* per ell *Flemish*, for 5 months; *B.* 600 guineas for 3 months; and *C.* 1800 l. for 4 months; but at 3 months end, takes out 800 l. they gain in all 1000 l. How much is each of their shares?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	860	4	3 $\frac{1}{2}$	3996
{ <i>B.</i>	61	11	8	1920
{ <i>C.</i>	78	4	0 $\frac{1}{4}$	2568



ARTS AND CRAFTS

CHAP. IX.

Of ALLIGATION.

ALLIGATION is the compounding many simples into one mass, according to any requir'd price, or proportion.

This rule is usually divided into two parts, distinguished by the names of *Medial* and *Alternate*.

ALLIGATION MEDIAL.

Alligation Medial answers such questions as having the quantities and prices of the several simples given, only demand the mean rate of any particular part of the composition.

R U L E.

As the sum of all the simples,
To their total value;
So is any quantity of the mixture,
To the price thereof.

E X A M P L E S.

(Ex. 1st.) A Grocer would mix 18 lb. of currants at 5 d. per lb, and 29 lb. at 6 d. per lb, with 12 lb. at 8 d. per lb, At what rate may he sell a pound of the mixture?

First,

First, place the quantities and their values, as below.

lb.	d.	s.	d.
18	at 5 per lb. comes to	7	6
29	at 6 <u> </u> to	14	6
12	at 8 <u> </u> to	8	0

Sum of all the Sim. 59 l. 1 10 0 tot. val.

Then, If 59 comes to l. 1 10 what will 1 ?

$$\begin{array}{r}
 20 \\
 \hline
 30 \\
 12 \\
 \hline
 59 \overline{) 360} \text{ (6 Answer.} \\
 \underline{354} \\
 6
 \end{array}$$

Ex. 2. A Grocer would with 12 C. of Sugar, at 1. per C. mix 3 C. at 50 s. per C. and 8 C. at 45 s. per C. What will 500 of this mixture be worth ?
The quantities and their values, placed as before, will stand thus :

C.	l.	s.
12 at 1 l. p. C. is	12	00
3 at 50 s. p. C. is	7	10
8 at 45 s. p. C. is	18	00

Sum of the Simples 23 61 10 tot. val.

R 6

Then,

[96]

Then, $\begin{array}{c} G. \\ 23 \end{array} \begin{array}{c} l. \\ 61 \end{array} \begin{array}{c} s. \\ 10 \end{array} \begin{array}{c} G. \\ 5 \end{array}$

$\begin{array}{r} 20 \\ \hline \end{array}$

$\begin{array}{r} 1230 \\ \hline \end{array}$

$\begin{array}{r} 5 \\ \hline \end{array}$

$23)6150(267$

$\begin{array}{r} 46 \\ \hline \end{array}$

$131. 7 s. 4 d. \frac{1}{2}. facit.$

$\begin{array}{r} 155 \\ \hline \end{array}$

$\begin{array}{r} 138 \\ \hline \end{array}$

$\begin{array}{r} 170 \\ \hline \end{array}$

$\begin{array}{r} 161 \\ \hline \end{array}$

$\begin{array}{r} 9 \\ \hline \end{array}$

$\begin{array}{r} 12 \\ \hline \end{array}$

$23)108(4$

$\begin{array}{r} 92 \\ \hline \end{array}$

$\begin{array}{r} 16 \\ \hline \end{array}$

$\begin{array}{r} 4 \\ \hline \end{array}$

$23)64(2$

$\begin{array}{r} 46 \\ \hline \end{array}$

$\begin{array}{r} 18 \\ \hline \end{array}$

The proof of this, is as in the *Rule of Three*, only by reversing the proportion.

So the first example may be prov'd by this by stating.

$\begin{array}{c} lb. \\ If 1 \end{array} \begin{array}{c} d. \\ 6 \end{array} \begin{array}{c} lb. \\ 59 \end{array}$

And the last by this,

$\begin{array}{c} G. \\ If 5 \end{array} \begin{array}{c} l. \\ 13 \end{array} \begin{array}{c} s. \\ 7 \end{array} \begin{array}{c} d. \\ 4 \frac{1}{2} \end{array} \begin{array}{c} G. \\ 23 \end{array}$

remembering in both to take in the remainders.

More EXAMPLES.

If a vintner mingleth 12 gallons of canary, at 6 s. per gallon, with 20 gallons of white wine, at 5 s. per gallon, and 10 gallons of cyder, at 3 s. per gallon, what rate must he sell a quart of the mixture?

Answer, 1 s. 3 d.

If with 40 bushels of corn, at 4 s. per bushel, there be mix'd 10 bushels at 6 s. per bushel, 30 bushels at 5 s. per bushel, and 20 bushels at 3 s. per bushel, what will 10 bushels of the mistling be worth?

Answer, 2 l. 3 s.

ALLIGATION ALTERNATE.

Alligation Alternate teacheth, what quantities of several simples (whose values are given) must be mix'd, that the composition may bear a price propounded.

This sort of *Alligation* has three varieties.

VARIETY I.

When the price of each simple is express'd, but no quantity given, and it is required how much of each simple must be mix'd, to sell any part of the composition at a mean price propounded.

In this variety you have only to link the several extreams rightly together, and to find the true differences betwixt them and the mean; for those differences are the quantities sought.

EXAMPLE.

A vintner would mix four sorts of wine of 12 d. of 18 d. of 24 d. and of 36 d. per quart, what quantity of each must he take, to sell the mixture at 20 d. per quart?

To

To place your numbers, and link the extreame observe these rules.

R U L E S.

1st, Set down the several given prices (which must always be of one name) in a column, one under another; then, towards your left hand, draw a line of connection; and on the other side of the said line, place the mean rate.

Thus then will stand the given numbers of the given example.

$$\begin{array}{c} 12 \\ 18 \\ 20 \\ 24 \\ 26 \end{array}$$

2^d, Link them two and two together, always observing to join a greater and a less than the mean, and against each extreame place the difference betwixt the mean and its yoke-fellow:

Thus,

$$\begin{array}{c} 12 \\ 18 \\ 20 \\ 24 \\ 26 \end{array} \begin{array}{c} 4 \\ 6 \\ 8 \\ 2 \end{array}$$

So the difference betwixt the extreame 12, and 20 the mean, being 8, is placed against 24 its yoke-fellow; and between 18 and 20 being 2, is placed against 26: So also the difference betwixt 24 and 20 being 4, is placed against 12; and 6, the difference betwixt 26 and 20, is set against 18.

Therefore if he mixes 4 quarters at 12 *d.* 6 at 18 *d.* 8 at 24 *d.* and 2 at 26 *d.* per quart, he may sell the mixture at 20 *d.* per quart.

The proof of which is easy. For the sum of the differences valu'd at the mean rate, will be found equal to the amount of the particular differences at their given prices.

For

For 20 quarts, the sum of the differences at 20 *d.* per quart, is 400 pence; and 4, 6, 8, 2 quarts, the particular differences, at 12, 18, 24, 26 pence per quart will come to the same sum.

But note, as many different ways as the extreame may be combin'd, so many different answers may be given to the question, yet all true:

So the same example being combin'd

thus:

$$\left\{ \begin{array}{r} 12 \\ 18 \\ 24 \\ 26 \end{array} \right\} \begin{array}{|c} \hline 6 \\ \hline 4 \\ \hline 2 \\ \hline 8 \\ \hline \end{array} \right\} \text{Ans.}$$

or thus:

$$20 \left\{ \begin{array}{r} 12 \\ 18 \\ 24 \\ 26 \end{array} \right\} \begin{array}{|c} \hline 4.6.10 \\ \hline 4.6.10 \\ \hline 8.2.10 \\ \hline 8.2.10 \\ \hline \end{array} \right\} \text{Ans.}$$

we have those different, but true answers, as may be prov'd by the same rule.

In the last of these examples the numbers being doubly combin'd, (*i. e.*) each with two others, make each have two differences set against it; for 12 the first extreame, being link'd both with 24 and 26, must have both their differences, *viz.* 4 and 6, plac'd against it; and so the rest. Which double difference must be added, and plac'd as above.

But when in a question there is but one extreame, or but one greater than the mean; such a question will admit of but one answer: that single extreame being to be link'd with all the rest.

EXAMPLE.

A grocer would mix sugars, at 5 *d.* 6 *d.* and 10 *d.* per lb, so as to sell the compound at 8 *d.* per lb; what quantity of each must he take?

$$8 \left\{ \begin{array}{r} 5 \\ 6 \\ 10 \end{array} \right\} \begin{array}{|c} \hline 2 \\ \hline 2 \\ \hline 3.2 \\ \hline \end{array} \left| \begin{array}{c} 2 \\ 2 \\ 5 \end{array} \right.$$

The numbers being plac'd, link'd, and differenc'd, as above) according to the foregoing rules, it appears to sell his sugar at 8 *d.* per lb, he must mix 2 lb. at 5 *d.* 2 lb. at 6 *d.* and 5 lb. at 10 *d.* per lb.

The

The reason of these combinations, and the alternate placing of their differences, will appear from this plain consideration, *viz.* That thereby whatever is lost upon the quantity fold, whose given price exceeds the mean, is gain'd again upon the quantity, whose given price is less than the mean.

More *EXAMPLES.*

A tobacconist would mix tobacco of 2*s.* and 1*s.* 6*d.* and 1*s.* 3*d.* per lb, so as to sell the compound at 1*s.* 8*d.* per lb, what quantity of each must he take?

lb. *s.* *d.*

Answer, $\left\{ \begin{array}{l} 7 \text{ at } 2 \text{ } 0 \\ 4 \text{ at } 1 \text{ } 6 \\ 4 \text{ at } 1 \text{ } 3 \end{array} \right\}$ per lb.

Several sorts of tea, *viz.* of 13*s.* 18*s.* 26*s.* and 30*s.* per lb, are to be so mix'd, that the compound may be worth 22*s.* per lb, what quantity of each must there be?

lb. *s.*

Answer, $\left\{ \begin{array}{l} 8 \text{ at } 13 \\ 4 \text{ at } 18 \\ 4 \text{ at } 26 \\ 9 \text{ at } 30 \end{array} \right\}$ per lb.

VARIETY 2.

Here we have the price of all the simples, and the quantity of one given, to find the several quantities of the rest: so that one measure, or quantity may be sold at a mean rate propounded.

When the numbers have been set down, and the several differences found by the foregoing rules, you must proceed by this proportion.

As the difference of that number whose quantity is given,

To the rest of the differences one after another:
So the quantity given,

To the several quantities sought.

EXAMPLE

EXAMPLE.

A distiller would with 40 gallons of *French* brandy, at 12 s. per gallon, mix *English* brandy at 7 s. per gallon, and spirits at 4 s. per gallon; how much of the two last sorts must be added, to sell the whole at 8 s. per gallon?

$$8 \left\{ \begin{array}{l} 12 \\ 7 \\ 4 \end{array} \right. \begin{array}{l} \boxed{1} \\ \boxed{4} \\ \boxed{4} \end{array} \begin{array}{l} 1.4 \\ 4 \\ 4 \end{array} \left| \begin{array}{l} 5 \\ 4 \\ 4 \end{array} \right.$$

Now it is plain, that were there but 5 gallons of the *French* brandy, there must also be but 4 gallons of each of the other liquors; but since there are to be 40 gallons of the *French* brandy, say,

As 5, the difference against its price,

To 4, the difference against the price of the *English* brandy :

So is 40, the given quantity of the *French*,

To the quantity of *English* sought,

which by this proportion will be found to be 32.

Then again,

As the first difference 5,

To 4, the difference against the price of the spirits :

So 40, the said given quantity,

To the quantity of spirits sought.

Which because their differences are alike, will be the same as of the *English* brandy, viz. 32.

This variety has the same proof as the former.

More EXAMPLES.

How much tea at 14 s. per lb, and of 18 s. per lb, must be mix'd with 8 lb, at 24 s. per lb, so as to allow the lb at 20 s?

$$\text{Answer. } \left\{ \begin{array}{l} 8 \text{ lb. at } 24 \text{ s.} \\ 4 \text{ lb. at } 18 \text{ s.} \\ 4 \text{ lb. at } 14 \text{ s.} \end{array} \right.$$

I would with 21 lb of snuff, at 8 s. per lb, mix so much of 4 s. per lb, and so much of 5 s. per lb, that I might

might afford to sell the mixture at 6 s. per lb; what quantity of each must I take?

Answer, $\left\{ \begin{array}{l} 21 \text{ at } 8 \\ 14 \text{ at } 4 \\ 14 \text{ at } 5 \end{array} \right\} \text{ per lb.}$

VARIETY 3.

In this variety, the particular prices of each simple being given, as also the mean rate of the compound, it is required how much of each sort must be taken, to make up the quantity propounded?

Which (after the work of the first variety) is performed by this proportion:

As the sum of the differences
To the total quantity:
So each particular difference
To its particular quantity.

EXAMPLE.

A druggist hath 4 sorts of green tea, at 5 s. 6 s. 8 s. and 9 s. per lb, and would make up a quantity of 87 lb to sell at 7 s. per lb, how much of each must he take?

See the work.

	lb.	s.	
$\left\{ \begin{array}{l} 2 \\ 6 \\ 8 \\ 9 \end{array} \right\}$	2—29	at 5 per lb.	
	1—14 $\frac{1}{2}$	at 6 per lb.	
	1—14 $\frac{1}{2}$	at 8 per lb.	
	2—29	at 9 per lb.	
		6	

lb.	lb.	lb.
If 6—	87—	
	2	
		lb.
	6)174	(29
	0	
lb.	lb.	lb.
If 6—	87—	
	1	
		lb.
	6)87	(14
	3	

More *EXAMPLES*.

A vintner hath three sorts of wine, of 6*s.* 8*s.* and 10*s.* per gallon, of all which he would compose quantity of 120 gallons, to sell at 7*s.* per gallon, what quantity must there be of each?

$$\begin{array}{r} \text{gall.} \quad \text{s.} \\ \text{Answer, } \left\{ \begin{array}{l} 80 \text{ at } 06 \\ 20 \text{ at } 08 \\ 20 \text{ at } 10 \end{array} \right\} \text{ per Gal.} \end{array}$$

A grocer having sugars of 5*d.* 6*d.* 8*d.* 10*d.* and 12*d.* per lb. would make a mixture of 3 C. wt. to sell at 7*d.* per pound, how much of each must he take?

$$\begin{array}{r} \text{lb.} \quad \text{d.} \\ \text{Answer, } \left\{ \begin{array}{l} 28 \text{ at } 05 \\ 196 \text{ at } 06 \\ 56 \text{ at } 08 \\ 28 \text{ at } 10 \\ 28 \text{ at } 11 \end{array} \right\} \text{ per lb.} \end{array}$$





CHAP. X.

Of POSITION, or the Rule FALSE.

THIS rule has not the name of *False* from being in its self really erroneous, but only because that by the help of false suppos'd numbers we find the truth.

It is usually divided into two parts, *single* and *double*.

Single POSITION.

IN *Single* POSITION we make but one supposition, with which working as with a true number, we regulate the result by this proportion, viz

As the total arising from the error,
To the true total;
So the suppos'd part,
The true one.

EXAMPLES.

1st, *A. B.* and *C.* designing to buy a quantity of lead to the value of 140*l*. agree that *B.* shall pay as much again as *A.* and *C.* as much again as *B.* what then must each pay?

Now

Now
1. an
ould b
If 70

and t

2. Su
and C
ordin

it sho
uld be

1.
If 21

ich, r
nd by
th

by

A gen
r 50*l*.

Now, supposing *A.* to pay 10*l.* then *B.* must pay 1*l.* and *C.* 40*l.* the total of which is 70*l.* but should be 140*l.* therefore,

If 70*l.* should be 140*l.* what should 10*l.* be?

Answer, 20 for *A.*'s share,
which doubled, makes 40 for *B.*'s share,
and that again doubled, gives 80 for *C.*'s share.

The total of which is 140

2. Suppose the sum of 20*l.* were to be paid by *A.* and *C.* thus, *A.* $\frac{1}{2}$, *B.* $\frac{1}{3}$, *C.* $\frac{1}{6}$, what must each pay according to that proportion?

	l.	s.	d.
First, the $\frac{1}{2}$ of 20 being	10	00	0
the $\frac{1}{3}$	6	13	4
and the $\frac{1}{6}$	5	00	0

The total will be 21 13 4

it should have been but 20*l.* therefore the parts found should be thus regulated:

	l.	s.	d.
l. s. d.	10	00	0
If 21 13 4 fall to 20,	06	13	4
what will	05	00	0

which, with the addition of $\frac{1}{6}$ makes just 20 00 0—
and by dividing the total of 28
the remainders, 36
40

by the common divisor 52)104(2

More EXAMPLES.

A gentleman bought a chaise, horse, and harness for 50*l.* the horse came to twice the price of the harness,

harness, and the chaise to twice as much as horse and harness, what did he give for each ?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainder</i>
<i>Answer,</i> { the harness	05	11	1	$\frac{1}{2}$ ——— 15
the horse	11	02	2	$\frac{1}{2}$ ——— 30
the chaise	33	06	8	—————

A. B. and *C.* disputing of their age, *A.* affirms is as old as *B.* and $\frac{1}{2}$ as old as *C.* and *B.* says, that is $\frac{1}{2}$ as old as *C.* upon which *C.* says, he is sure be their ages added to his, will make 110 years. I mand the age of each ?

<i>Answer,</i> {	<i>A.</i>	45	$\frac{1}{2}$
	<i>B.</i>	37	$\frac{1}{2}$
	<i>C.</i>	36	$\frac{1}{2}$

Double POSITION.

IN *Double POSITION* two suppositions are quir'd, which (if both prove false) are, w their errors, to be thus dispos'd.

Set down both your suppositions, and against of them their respective errors, mark'd thus — too much, or thus —, if too little.

Then multiplying them cross ways, that is, first supposition by the second error, and the second supposition by the first error, if both the errors alike, *i. e.* both too little, or both too much, subtract the lesser product from the greater, and divide the remainder by the difference of the errors. If the errors are unlike, *i. e.* one too little, and the other too much, the sum of these products must be divided by the sum of the errors: the quotient will give the true number, exactly answering all the demands of the question.

Or the suppositions and their errors, being plac'd before, work by this proportion, as a general rule,

As the difference of the errors, if alike,
Or their sum, if unlike,
To the difference of the suppositions :

So either error to a fourth number,
Which accordingly added to, or subtracted from,
The supposition against it, will answer the question,

EXAMPLES.

1st, *A. B. and C. playing at cards, the money
mark'd was 324 crowns; but disagreeing, each seiz'd
as many as he could; A. got a certain number, B.
as many as A. and 15 over; C. got a fifth part of both
their sums added together, how many had each?*

First, suppose *A. got 50*
then *B's share will be 65*
and *C's* ————— *23*

Which added, make but 138 which is 186 too few!

Then, supposing again that *A. got 80*
B's share will be 95
and *C's* ————— *35*

The total of which is still but 210, too few by 114.

Therefore, setting down the suppositions, and
their errors (mark'd as before directed) against them,
they will stand thus :

Suppositions.	Errors.
50 —————	186
80 —————	114

When multiply'd cross-ways, the products will be
890, and 5700; the lesser of which (the errors
being alike, &c. both too little) being taken from
the greater, there will remain 9180 for a dividend,
which divided by 72, the difference of the two er-
rors, quotes 127 $\frac{1}{2}$, the true number of crowns *A.*
took up; as will appear, by adding 15 to them for
B's share, and dividing the sum of *A's* and *B's* by 5

for
†

for C's share; for so their particular numbers will be found to be as follow :

$$\begin{array}{r} A's \text{ --- } 127 \frac{1}{2} \\ B's \text{ --- } 142 \frac{1}{2} \\ C's \text{ --- } 54 \\ \hline \end{array}$$

The total of which is 324, the number of crowns given

Or by the other rule,

As 72, the difference of the errors, (because alike to 30, the difference of the suppositions :

$$\text{So is } \left\{ \begin{array}{l} 186 \\ \text{or} \\ 114 \end{array} \right\} \text{ to } \left\{ \begin{array}{l} 77 \frac{1}{2} \\ 47 \frac{1}{2} \end{array} \right\} \text{ which added } \left\{ \begin{array}{l} 50 \\ \text{or} \\ 80 \end{array} \right\} \text{ gives } 127 \frac{1}{2} \text{ before.}$$

EXAMPLE 2.

When first the marriage-knot was ty'd
Betwixt my wife and me;
My age did hers as far exceed,
As three times three does three.
But after ten and half ten years,
We man and wife had been;
Her age came up as near to mine,
As eight is to sixteen.

Now, pray,
What were our ages on the wedding-day?

First, suppose her 21 then he must be 63
then adding to each 15 15
her ages becomes 36 his 78

This error therefore is 3 years too few.

But supposing her 9, the error will prove 3 too many.

Setting down therefore against the suppositions the errors, with their respective marks, they will stand thus :

Supp

Suppositions. Errors.

$$\begin{array}{r} 21 \quad \text{---} \quad 3 \\ 9 \quad \text{+} \quad 3 \\ \hline \end{array}$$

Then multiplying cross-ways, 3 times 21 is 63
3 times 9 is 27

These products (because unlike) added, make 90
a dividend, which divided by 6, the sum of the
errors, quotes 15, her true age, prov'd thus:

$$\begin{array}{r} \text{She being } 15, \text{ he must be } \text{---} 45 \\ 15 \text{ years added to each } \text{---} 15 \\ \hline \end{array}$$

Makes her 30, just half of his --- 60

Or, by the other rule,

6, the sum of the errors, (because unlike) to
the difference of the suppositions:

3 is the common error, 3 to 6, which either
tracted from 21, or added to 9, gives 15, (as be-
fore) for her age.

MORE EXAMPLES.

What number is that, to which if you add $\frac{1}{4}$ of
itself, and from the sum subtract $\frac{1}{3}$ of its self, the
remainder will be 210?

Answer, 229.

Gentleman caught a fish, whose head was 6 in-
ches long, the tail as long as the head, and half as
long as the body; the body was just the length of
the head and tail. I demand the length of the fish?

Answer, 48 inches, or 4 foot.



C H A P. XI.

Of E X C H A N G E.

EXCHANGE is the receiving money in one Country, for the value paid in another.

N O T E.

The Par of exchange is certain and fix'd, but always *par pro pari, like for like*, according to weight and fineness of the coin.

But the Course or current running price, betwixt any two countries, rises and falls upon every occasion.

It would be both needless and endless to write of every kind of exchange; I shall therefore only give examples of the exchanges of England with some other chief countries of Europe, first with

F R A N C E.

At *Paris, Lyons, Rouen, &c.* they keep their accounts in *livres, sols, and deniers*; and exchange upon the crown, the par of which in sterling money, is 4*s.* 6*d.*

N O T E.

12 Deniers }
20 Sols } make { 1 Sol.
3 Livres } { 1 Livre.
 } { 1 Crown.

French money is chang'd into *Sterling* by this proportion.

As a crown to the given rate;

So is the given *French* sum to the *Sterling* sought.

Or,

By supposing the given *French* crowns, *soles*, and rate of exchange, the *price*; so casting up their value by practice.

E X A M P L E S.

1. What *Sterling* money must be paid in *London*, to receive in *Paris* 430 crowns; exchange at 56 s. per crown?

£ 56 — 430 56 ----- 2688 127 14 12 ----- 3190 13) 34528 ----- 210) 30414 ----- 102 7. 4 s.	£ 56 at 56 s. or 4 l. 8 s. per Cr. ----- 430 127 14 12 ----- 102 4
---	---

£ 2

2d. Change

2d. Change 537 crowns, 14 fols, 10 deniers, into
Sterling money, exchange at 52 d $\frac{1}{2}$. per crown.

Cr.	d.	Cr.	Sols	De.	Cr.	Sols	De.	d.
If 1	52 $\frac{1}{2}$	537	14	10	537	14	10	at 52
60	2	60						

12	105	32234	4s. $\frac{1}{2}$	107	8
720		12	4d. $1\frac{1}{2}$	8	19
			$\frac{1}{2}$	1	2
			10	4	$\frac{1}{2}$
			2	1	$\frac{1}{2}$
			2	1	$\frac{1}{2}$
			Deniers 10 $\frac{1}{2}$	of 10 Sols.	

1034050	
3868180	117 l. 10s. 5d.

72 0) 4061589 0	(56410
360	
461	13) 18105 (5
433	2 0) 135 0
395	
388	117 l. 10s. 5d $\frac{1}{2}$ facit.

78
72
69
2
72) 138 (1
72
66

Sterling money is chang'd into *French* by this proportion :

As the rate of exchange, to one crown :

So is the given *English* sum, to the *French* sought.

For EXAMPLE.

Take the reverse of the former questions ; which will also be their proof.

Q. How many *French* crowns must be paid in Paris, to receive in London 102 l. 4 s. *Sterling* ? Exchange at 56 d. per crown.

d.	Gr.	l.	s.
If 56	— 1 —	102	4
		20	
		2044	
		12	
		CROWNS.	
		56	24528 (438
		224	
		212	
		168	
		448	
		448	
		0	

2d. Change 117 l. 10 s. 5 d $\frac{1}{2}$. Sterling, into French crowns. Exchange at 52 d $\frac{1}{2}$. per crown.

d. Crown. l. s. d.
If 52 $\frac{1}{2}$ ——— 117 10 5 $\frac{1}{2}$

4
210

20
2350
12

28205

51
60
Sols.

21 | 0) 306 | 0 (14

21

6

84

12

12

144

66 the remainder of the other sum.

21) 216 (10 deniers.

21

00

More EXAMPLES.

If I pay in Paris 8297 crowns, 12 sols, 8 deniers what may I draw my bill for to London, when the exchange is at 54 d $\frac{1}{2}$. per crown?

Answer, 1875 l. 10 s. 4 d $\frac{1}{2}$.

What Sterling money is equal to 954 crowns, sols, when the exchange is at 58 d $\frac{1}{2}$. per crown?

Answer, 231 l. 5 s. 1 d $\frac{1}{2}$.

If I pay in London 536 l. what French money shall I receive in Paris, exchange at 57 d. per crown?

Answer, 6893 crowns, 50 sols, 6 deniers.

What French money is equal to 1529 l. 10 s. exchange at 53 d. $\frac{1}{2}$ per crown?

Answer, 6893 crowns, 31 sols, 3 deniers.

Secondly, with ITALY.

In Genoa, Leghorn, &c. they keep their accounts in livres, sols, and deniers, and exchange upon a piece of eight, or dollar; the par of which with London, is also 4 s. 6 d. Sterling.

NOTE.

At Genoa 5 livres make a piece of eight, at Leghorn 6.

Observe the rules for exchanging with France.

EXAMPLES.

How much Sterling shall I receive in London, if I pay in Genoa 820 dollars; exchange at 51 d. per dollar, or piece of eight.

Bar.	d.	Dollars.	Dollars.
—	51	820	820 at 51 d. per dollar.

51	4
----	---

820 d.	3280
--------	------

4100	3 $\frac{1}{2}$	205
------	-----------------	-----

12)41820(	34815
-----------	-------

210)34815	174 l. 5 s.
-----------	-------------

174 l. 5 s.

F 4.

Or,

Or, back again;

If in London I pay 174 l. 5 s. Sterling, how many dollars shall I receive in Genoa? exchange at 51 per dollar.

d.	Dollar.	l.	s.
If 51	1	174	5
		20	

3485
12

51) 41820 (820
408

102
102

000

Denier exchanges by the ducat, the par at 56 d. $\frac{1}{2}$ Sterling.

N O T E.

6 Solidi } make 1 Gros
24 Groſſes } 1 Ducat.

E X A M P L E.

Change 512 ducats into Sterling-money, change at 55 d. $\frac{1}{2}$ per ducat.

D. d. D.
If 1 = 55 $\frac{1}{2}$ = 512 *

2 512
512

111 2) 56932(

12) 28416(

2|0) 236|8

118 l. 8 s.

Ducats. d.
512 at 55 $\frac{1}{2}$ per Duc.

4

d. 2048

6 $\frac{1}{2}$ 256

1 $\frac{1}{2}$ $\frac{1}{4}$ 64

236|8

118 l. 8 s.

* See Abbreviations of Multiplication.

Thirdly, with SPAIN.

In Madrid, Sevil, &c. they keep their accompts in rials and mervadies, and exchange also by the piece of eight. The par with London 4s. 6d.

N O T E.

372 Mervadies } make § 1 Rial.
8 Rials } 1 Piece of §

This exchange is also cast up by the foregoing rules.

E X A M P L E.

What *Sterling* money may I draw for to *London*, if I pay in *Sevil* 4768 rials, 250 mervadies? exchange at 58 d. *Sterling* per piece of §.

P s

Rials.

[118]

Rials.	d.	Rials.	Mer.
If 8	58	4798	250
372		372	

2976

9596

33591

14396

1785106
58

14280848
8925530

2976) 103536148 (12
8928

14356 210) 28919 (

11904 144 19 2 4

23521
20832

26894
26784

1108
4

2976) 4432 (1
2976

1456

Cha
ey. E

0

Or, back again,

Change 144 l. 19 s. 2 d. $\frac{1}{4}$. Sterling into Spanish money. Exchange at 58 d. per piece of $\frac{1}{8}$.

d.	Rials.	l.	s.	d.
158	8	144	19	2 $\frac{1}{4}$
4			10	
232		2899		
		12		
		34790		
		4		
		139161		
		8		
232	1111288	(4798 Rials.		
	928			
	1852			
	1624			
	2288			
	2088			
	2000			
	1856			
	152			
	372			
	204			
	1064			
	456			
	1456	Rem. of the other.		
232	58000	(250 Mervadies.		
	464			
	1160			
	1160			
	0			

E 6.

Mere

More E X A M P L E S.

Change 83927 *Rials* into pounds *Sterlings*, exchange at 54 $d \frac{1}{2}$ per piece of eight.

Answer, 2382 *l.* 6 *s.* 0 $d \frac{1}{2}$.

If I pay in London 1000 *l.* what Spanish money may I draw my bill for to Madrid? exchange at 57 $d. \frac{1}{8}$ per piece of eight.

Answer, 168052 *Rials*, 192 *Mervadies*.

Fourthly, with PORTUGAL.

IN Lisbon, Oporto, &c. they keep their accounts in Rees, and exchange on the Mill-ree; the value of which is about 6 *s.* 8 $d \frac{1}{2}$ *Sterling*.

N O T E.

1000 Rees make 1 Mill- Ree.

To make this exchange, still observe the same rule.

E X A M P L E S.

Change 48009 rees into *Sterling* money, exchange at 6 *s.* 5 *d.* per mill-ree.

M. R.	s. d.	Rees.
If 1	6 5	48009
1000	12	77
	77	316063
		316063
		12
1/000	3696/693	(3696(
	4	
	2/0	30/9
	2/772	
		15 <i>l.</i> 8 <i>s.</i> 0 $d \frac{1}{2}$.

Or, back again,

Change 15 l. 8 s. 6 d. $\frac{1}{2}$ Sterling into rees, exchange at 6 s. 5 d. per mill-ree.

6 s. 5 d.	Rees.	l. s. d.
12	1000	15 8 6 $\frac{1}{2}$
		<u>20</u>
77		308
4		<u>12</u>
308		396
		<u>4</u>
		14786
		<u>1000</u>

Note, 772, Rem. of 308) 14786772 (41009
 the other Stating, it
 are taken in.

2466
<u>2464</u>
2
<u>2772</u>
2772
<u>0</u>

More EXAMPLES.

What Sterling money is equal to 595677 rees exchange, at 6 s. 7 d. per mill-ree?

Answer, 196 l. 1 s. 6 d. $\frac{1}{2}$.

Change 59 l. 12 s. into rees, exchange at 6 s. 7 d. $\frac{1}{2}$ per mill-ree?

Answer, 173381 rees.

Fifthly,

**Fifthly, with HOLLAND, FLANDERS,
and GERMANY.**

I Take these places together, because their accounts are kept, and their exchange with England made the same way.

For not only in *Amsterdam*, but also in *Antwerp* and *Hamborough*, they keep their accounts in pounds, shillings, and pence *Flemish*; or guilders, stivers, and pennicks; and exchange with us upon our pound, giving us for it, when at *par*, 33 s. 4 d. *Flemish*.

N O T E.

Their pounds, shillings, and pence, are divided as ours, *viz.* their pound into 20 shillings, and their shilling into 12 pence.

Their other money is thus divided.

16 Pennicks } make { 1 Stiver.
20 Stivers } make { 1 Guilder.

Note also,

6 Stivers } make { 1 Shilling } *Flemish*.
6 Guilders } make { 1 Pound }

Sterling money is chang'd into *Flemish* by this proportion, *viz.*

As 1 *l. Sterling* to the given rate :

So is the given *Sterling* to the *Flemish* sought.

Or,

By supposing the given *Sterling*, *goods*, and the given rate, their *price*, casting up their value by *practice*.

E X A M P L E

EXAMPLE.

IF I pay in London 492 l. Sterling, what may I draw my bill for to Amsterdam? exchange at 34 s. d. Flemish per pound Sterling.

l.	s.	d.	l.
492			492
13			419
Or by Practice.			1476
419			492
492	at 34	s per l.	1968
10 1/2			246
4 1/2			98 8
4 1/2			8 4
1 1/2			2 1
846 l. 13 s. facit.			846 l. 13 s. Flemish.

If you would have the answer in guilders and stivers,

Divide the *Flemish* pence by 40, (the number of pence which make a guilder) and the quotient will be guilders; and if any thing remains, the half of it will be stivers, two *Flemish* pence being one stiver.

Or,

Multiply the *Flemish* pounds, and shillings, by 20, and it will produce guilders and stivers; for guilders make 1 pound *Flemish*, and 6 stivers a shilling *Flemish*, and if there be any odd pence, multiply them by 8, for *Pennicks*.

EXAMPLE.

846 l. 13 s. the facit of the last sum.

6

Flemish pence.

4|0)20319|6(36 Rem.

5079 G. 18 S.

5079 Guild. 18 Stivers.

Flemish

Flemish money (whether pounds, shillings, or guilders, stivers, and pennicks) is exchanged into Sterling thus :

As the given rate to 1 pound Sterling : so is the given *Flemish* to the Sterling sought.

E X A M P L E S.

Change 846 l. 13 s. *Flemish* into Sterling money, exchange at 34 s. 5 d. *Flemish* per l. Sterling.

s.	d.	Flem.	l.	Sterl.	l.	s.	Flem.
If	34	5	—	1	—	846	13
	12					20	

413

16933

12

413) 203196 (492 l. Sterling
1652

3799

3717

826

826

0

Or,

Change 5079 guilders, 18 stivers into Sterling money, exchange 34 s. 5 d. per l. Sterling.

s. d. Flem. l. Ster. Guild. Stiv.
 If 34 5 ——— 1 ——— 5079 18

12 ——— 20
 413 ——— 101598
 2

413) 203196 (492 l. Sterl.
 1652

3799
 3717

826
 826

0

More EXAMPLES.

Change 475 l. Sterling into *Flemish*-money; exchange at 33 s. *Flemish* per l. Sterling?

Answer, 783 l. 15 s. Flemish.

What *Flemish* money is equal to 365 l. 12 s. exchange at 36 s. 5 d. per l. Sterling?

Answer, 665 l. 13 s. 11 d. Flemish.

If I pay at *Amsterdam* 5389 l. 14 s. *Flemish*, what Sterling money shall I receive at *London*, exchange at 32 s. 8 d. *Flemish* per l. Sterling?

Answer, 3299 l. 16 s. 3 d. $\frac{1}{4}$.

Change 572 l. 13 s. Sterling into guilders, stivers, &c. Exchange at 34 s. 10 d. *Flemish* per l. Sterling.

Answer. 5984 guilders, 3 stivers, 13 pennicks.

What may I draw my bill for to *London*, if I pay in *Antwerp* 7256 guilders exchange, at 35 s. 2 d. $\frac{1}{2}$ per l. Sterling?

Answer, 686 l. 19 s. 2 d. Sterling.

AN

*An Explanation of some Characters sometimes
us'd for Brevity's Sake.*

$+$ Is the mark of *Addition*, and shews the numbers
it's plac'd between, are to be added to
ther.

$-$ Is the mark of *Subtraction*, and shews the latter
of the numbers it stands between, is to be taken
from the former.

$\times$ Is the mark of *Multiplication*, and shews the
numbers on each side, are to be multiply'd together.

$\div$ Is the mark of *Division*, and plac'd between two
numbers, shews the first is to be divided by the last.

$=$ Is the mark of *Equality*, and shews the num-
ber or numbers preceding it, are equal to that which
follows it.

$\sqrt{\quad}$ Plac'd before any figure, shews the Square
Root of them is meant. And

$\sqrt[3]{\quad}$ before a number, denotes its Cube-Root.

$::$ Is the sign of *Proportion*, and being plac'd be-
twixt the two middle terms of 4 proportionals,
signifies, *so is*.

To make all more plain, observe,

Thus the Characters are us'd :

$$2 : 4 :: 6 : 12 \quad \sqrt[3]{4096} = 16, \sqrt{16} = 4, 4 + 2 = 6 \\ 6 - 2 = 4, 4 \times 2 = 8, 8 \div 2 = 4.$$

And thus read :

As 2 is to 4, so is 6 to 12 ; the Cube-Root of
4096 is, or is equal to, 16 ; the Square Root of 16, is
equal to 4 ; 4 added to, or more by 2, is equal to 6 ;
and 6, made less by 2 is equal to 4 ; again, 4
multiply'd by 2, is equal to 8, and 8 divided by
2, is equal to 4.

C H A P. XII.

of Arithmetical and Geometrical PRO-
PORTION, or PROGRESSION.

PROGRESSION being a rule more curious than useful, and requiring more words for its full application, than are consistent with this *Compendium*, I shall only give some (the most necessary) propositions in both its parts, arithmetical and geometrical; and refer the more inquisitive reader to Mr. Oughtred's *Clavis Mathematica*, for his farther satisfaction.

In all continued proportion, or comparison of numbers, that number which is compared to another is called the *antecedent*, and the number to which it is compared is called the *consequent*.

There are two ways of comparing numbers with one another. 1st. With respect only to their simple difference, *i. e.* how much the one, *viz.* the antecedent, is greater, or less than the other, *viz.* the consequent, found by subtraction; thus the difference betwixt 5 and 8, is 3; which difference is called the *Arithmetical Ratio*. 2^d. Numbers are compared with one another, when we consider how often one number contains, or is contained in another; and this is found by division, the quotient being the *geometrical ratio*. Thus, 2 is the *geometrical ratio* betwixt 8 and 16.

ARITHMETICAL PROGRESSION

ARITHMETICAL PROGRESSION is the regular increase or decrease of any rank of numbers by the continual addition or subtraction of some equal number.

So 1, 4, 7, 10, 13, 16, are in *Arithmetical Progression* to each other, equally increasing by the continual addition of 3.

And so also are 84, 82, 80, 78, regularly decreasing by the continual subtraction of 2.

Some of the most considerable properties of numbers in *Arithmetical Progression* are as follow :

1. That in any rank of numbers in *Arithmetical Progression* consisting of three, or any odd number of terms, the double of the middle term will be equal to the sum of the two extremes, or of any two means equally distant from the said middle term.

1, 2 3, 4, (5) 6, 7, 8, 9.

Thus in the rank above 5, the middle term doubled is 10, = to $1 + 9$, the two extremes, or $2 + 8$, or $3 + 7$. or $4 + 6$, the several correspondent means.

2. That in any rank of numbers in *Arithmetical Progression* consisting of four, or any even number of terms, the sum of the two extremes will be equal to the sum of the two middle numbers, or of any two means equally distant from the said extremes.

2, 4, 6, 8, 10, 12, 14, 16.

Thus in the rank above $2 + 16$, the two extremes are 18, = $8 + 10$; the two middle numbers or $6 + 12$, or $4 + 14$, the other correspondent means.

PROG

Any th
y be
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Or the
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6.

PROGRESSION, *five Things are to be noted,*

V I Z.

- I. The first Term.
- II. The last Term.
- III. The number of Terms.
- IV. The equal Difference.
- V. The Sum of all the Terms.

Any three of which being given, the other two may be found; as Mr. Oughtred in his *Clavis Mathematica*, Chap. 19. *Prob.* the 6th, exemplifies in 20 positions; some of the most useful of which as follow.

PROPOSITION I.

The last term, number of terms, and equal difference being given, to find the first term:

Or the second, third, and fourth given, to find first.

R U L E.

Multiply the fourth by the third, made less by 1, and subtract the product from the second, remainder is the answer.

E X A M P L E.

A man takes out of his pocket, at 6 several times, several numbers of shillings, every one exceeding the former by 6, the last was 42, what was first? *Answer*, 12s.

$$6 \times 5 = 30, 42 - 30 = 12 \text{ Answer.}$$

PROPO.

PROPOSITION II.

The first, third, and fourth given, to find the second.

R U L E.

From the product of the fourth, multiply'd by the third, subtract the fourth, the remainder added to the first, gives the second.

E X A M P L E.

What's the last number of an *Arithmetical Progression*, beginning at 4, and continuing, by the increase of 12 to 18 places? *Answer*, 208.

$$12 \times 18 = 216, 216 - 12 = 204, 204 + 4 = 208 \text{ Answer}$$

PROPOSITION III.

The first, second, and fourth given, to find the third.

R U L E.

Subtract the first from the second, and divide the remainder by the fourth, the quotient, with one added, gives the third.

E X A M P L E.

A traveller went 12 miles the first day, and increased every day's journey by 4 miles, till at last he went 64 miles in one day; How many days did he travel? *Answer*, 14 days.

$$64 - 12 = 52, 52 \div 4 = 13, 13 + 1 = 14 \text{ Answer.}$$

PROPOSITION IV.

The first, second, and third given, to find the fourth.

1

R U L E

R U L E.

Subtract the first from the second, and divide the remainder by the third, made less by one, the quotient gives the fourth.

E X A M P L E.

The ages of seven children increase by *Arithmetical Progression*; the youngest is two years old, the eldest 32, what's the common difference of their ages? *Answer*, 5 years.

$$32 - 2 = 30, 30 \div 7 - 1 = 5 \text{ Answer.}$$

P R O P O S I T I O N V.

The first, second, and third given, to find the fifth.

R U L E.

Half the sum of the first and second, multiply'd by the third, produces the fifth.

E X A M P L E.

12 persons give their charity to a poor man in *Arithmetical Progression*; the first gave 2 pence, the last 2 shillings, or 24 pence; how much did the poor man get? *Answer*, 156 pence, or 13 shillings.

$$2 + 24 = 26, 26 \div 2 = 13, 13 \times 12 = 156 \text{ Answer.}$$

G E O M E T R I C A L P R O G R E S S I O N.

GEOMETRICAL PROGRESSION is the increasing or decreasing of any rank of numbers by some equal *Ratio*, that is, by the continual multiplication or division of some equal number.

Thus,

Thus, 3, 4, 8, 16, 32, Increase in *Geometrical Progression* by a double *Ratio*, or continual Multiplication by 2, and 405, 135, 45, 15, 5, decrease after the same manner by a continual Division by 3, or by a triple *Ratio*.

Some of the most considerable properties of numbers in *Geometrical Proportion*, are as follow :

1. That in any rank of numbers in *Geometrical Progression*, consisting of three, or any odd number of terms, the product of the middle term multiplied by its self, will be equal to the product of the two extreams, or of any two means equally distant from the said middle term.

1, 2, 4, (8) 16, 32, 64.

Thus in the rank above, 8, the middle term squar'd, *i. e.* multiply'd by its self, is 64, $= 1 \times 64$, the two extreams, or 2×32 , or 4×16 , the correspondent means.

2. That in any rank of numbers in *Geometrical Progression*, consisting of four, or any even number of terms, the product of the two extreams, will be equal to the product of the two middle numbers, or of any two means equally distant from the said extreams.

2, 4, 8, 16, 32, 64.

Thus in the rank above 2×64 , the two extreams produces 128 $= 8 \times 16$, the two middle numbers, or to 4×32 , the other correspondent means.

In *Geometrical Progression*, the same five things are to be noted as in *Arithmetical Progression*.

V I Z.

- I. The first Term.
- II. The last Term.
- III. The number of Terms.
- IV. The *Ratio*, or equal difference.
- V. The Sum of all the Terms.

Note,

Note, Figures are sometimes set over a rank of geometrical proportionals as hereunder, and are called Indices, or Exponents, serving to shew the distance of any term from unity, or from the first term of the series.

Thus $\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 4 & 8 & 16 & 32 & 64 \end{array}$

PROPOSITION I.

To find any remote term of a *Geometrical Progression* proceeding from unity, whose *ratio* or common difference is known, without producing all the intermediate terms.

R U L E.

Find some few of the leading terms, and place over them their Exponents; then multiplying the first found term by its self, it will produce a term double thereto; which again multiply'd by its self, will produce another double to the last: Thus proceeded, till either you produce the term sought, or be a little short of it; which may be compleated by multiplying it again by that term, which stands under such exponent as would make up the number. Or in other words, observe what figures of the exponents found added together, would give the exponent of the term wanted, and the numbers standing under the said exponents, multiply'd into each other, will produce the term requir'd.

From whence may be observ'd the coherence or agreement betwixt numbers in arithmetical and geometrical proportion.

EXAMPLE.

One agrees, for 14 oranges to pay only the price of the last, at a farthing for the first, an half-penny the second, &c. still doubling the price for the next, what must he give?

G

Here

Note,

Here setting down only the five first terms, with their exponents, the fourteenth may thus be found $5 + 5 + 4 = 14$. So the terms under those exponents being multiply'd into each other, give the term answering. But as the exponent 1 stands always over the second term, so the exponent 13 answers to the 14 term; therefore add only 3 to the two 5's; and so always that exponent must be taken, which is one short of the term requir'd, as for 20, 19; for 30, 29, &c. which must be carefully noted.

See the work of the foregoing example.

0 . 1 . 2 . 3 . 4 . 5
1 . 2 . 4 . 8 . 16 . 32

Exponents.

5
5
3

13

Terms.

32 the 5th multiply
32 by its self.

64
96

gives 1024 the 10th, which
multiply'd by 8 the 3d Term,

gives 8192 the 14th.

And so many farthings, or 8 l. 10 s. 8 d. must give at the rate agreed.

PROPOSITION II.

To find any remote term of a Geometrical Progression, not proceeding from unity, whose ratio and first term is given, without producing all the intermediate terms.

AUL

R U L E.

Set down, as before, some few of the leading terms with their exponents, and multiply also, (as in the first proposition) the terms under such exponents, as added, would make the exponent short by one of the terms requir'd; only remember that every product must here be divided by the first term.

E X A M P L E.

A sum of money is thus to be divided among 9 persons, the 1st to have 50 l. the 2d 150 l. and so on in that proportion, one three times more than the other, to the last; what will the last have?

Setting down the five first terms with their exponents, thus,

0	1	2	3	4
50	150	450	1350	4050

Multiply the last term by its self, and divide the product by 50, the first term, the quotient will be 28050 the 9th term requir'd, answering to the 8th exponent.

Or setting down but the three first terms, multiply the last by its self, and divide the product by the first term, it quotes the 5th; which again multiply'd by its self, and divided by the 1st, will quote the 8th, as before. See the work.

1	2	450	4050
150	450	450	4050
22500		162000	
1800		162000	
510) 202500 (		510) 1640250 (	
4050		328050	

Answer, 328050 l. for the 9th person.

PROPOSITION. III.

Having the first term, the common excess, and the number of places given, to find the total sum of the whole progression.

R U L E.

Having found the last term by the foregoing rules, subtract from it the first, and divide the remainder by the common excess, made less by one; the quotient is the sum of all, except the last, to which, that being also added, gives the total required.

Note, If the *ratio* be double, that is, if the common excess be 2, the difference between the least and the greatest term added to the greatest, gives the total sum. If the *ratio* be triple, $\frac{1}{2}$ the difference must be added; if quadruple, $\frac{1}{3}$, &c.

E X A M P L E S.

1st, A coffee-man, upon the signing of the late Peace, for 50 guineas down, agreed to pay the first day one coffee-berry, the second two, the third four, and so on, to double the quantity every day till the same was proclaim'd. What number of berries would it amount to, supposing the time 30 days; and what would their value be, supposing 1000 berries to the pound, and the pound to be sold at 5s?

See the work.

[137]

Days. Berries.

1—1
2—2
3—4
4—8
5—16

16
16
96
16
256
2

512 the 10th day.

16
3072
512
8192
2

16384 the 15th day.

16384
65536

131072

49152

98304

16384

268435456

2

516879012 the 30th day.

516870912

1073741824

516870912

4831838208

37580963840

4294967296

3221225472

1610612736

2684354560

288230376151711744

2

576460752301423488 the 6th day.

1000 (11529215046068461975 Tot. number of Berries.

1. 288230376151711744 100. their val. at 5 s. per lb.

G 3

EN. 2.

Ex. 2. A grazier offers 40 oxen for a farthing a head, and treble it throughout ; to what sum will it amount ?

See the work.

Oxen. Farthings.

1 — 1	1162261467
2 — 3	1162261467
3 — 9	
4 — 27	8135830269
5 — 81	6973568802
81	4649045868
	1162261467
81	6973568802
648	2324522934
	2324522934
6561	6973568802
3	1162261467
	1162261467
19683 10th Ox.	
19683	1350851717672992089
	3
55049	
157464	4052555153018976267 40th Ox.
118098	
177147	6078832729528464400 Tot. Far.
19683	Or,
	6332117426592150 l. 8s. 4d.
387420489	
3	
1162261467 the 20th Ox.	

CHAP. XIII.

Of VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, and written with two figures, with a line drawn between them, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

The figure under the line is call'd the *denominator*, because it gives name to the fraction; and also shews into what parts the unit is broken; and that above the *numerator*, because it tells how many of such parts are meant by the fraction. Thus the fraction $\frac{3}{4}$ shews, that the unit is divided into 4 parts, and that 3 of those parts are thereby express'd.

1. Note therefore, That if the *numerator* and *denominator* of a fraction should be equal as $\frac{4}{4}$, $\frac{8}{8}$, the value of such fraction would be exactly an unit or integer; for, by the definition above, the *denominator* shews into how many parts the unit is broke; and the *numerator* expresses how many of those parts are meant by the fraction. Then in the fraction $\frac{4}{4}$, the *denominator* declaring the unit to be broke into 4 parts, and the *numerator* expressing 4, that is, all of those parts, it is plain the said fraction is equal to an unit or whole number; because the sum of all the parts must be equal to the whole. From which observation it is also plain, that so often as the *denominator* is contain'd in the *numerator*, so many whole numbers or units are contain'd in such improper

proper fraction. And this Note may serve as a reason for the operations of the 1st, 2d, and 3d sorts of Reduction.

2. Note also, That as all fractions do indeed arise from the remainders of divisions, when the divisor can no longer measure the dividend, so every fraction may be look'd upon as the two given terms of a division, the *numerator* as the dividend, and the *denominator* as the divisor; from whence it appears, that if the *numerator* and *denominator* of a fraction be either multiply'd or divided both by the same number, the products or quotients will still remain in the same proportion, and the new fraction so arising, be of the same value with that given. Thus the fraction $\frac{1}{2}$, multiply'd by 2, will produce $\frac{2}{4}$, or divided by 2, will quote $\frac{1}{4}$; all which fractions are of the same value; 4 bearing the same proportion to 8, and 1 to 2, as 2 does to 4. And from hence also may appear the reason of the 1st, 5th, and 6th sorts of Reduction.

Of Vulgar Fractions, there are four sorts, viz.

1st. A *proper*; whose *numerator* is always less than its *denominator*, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, &c.

2d. An *improper*; whose *numerator* is always equal to, or greater than its *denominator*, as $\frac{3}{2}$, $\frac{4}{3}$, &c.

3d. A *compound*, or *fraction of a fraction*; known by the word *of*, as $\frac{1}{2}$ of $\frac{1}{3}$, &c.

4th. A *mix'd fraction*; which is a fraction join'd with a whole number, as $5 \frac{1}{2}$, $3 \frac{3}{4}$, &c.

Before fractions can be either added or subtracted, they must be reduced into one denomination; and therefore, before we proceed to those rules, we must learn reduction.

REDUCTION OF FRACTIONS.

IN *reduction of fractions* there are seven varieties.

First, To reduce a whole number into an *improper fraction*, which is done by placing 1 for its denominator.

EXAMPLES.

Reduce 4 to a Fraction, *facit* $\frac{4}{1}$
or 18, $\frac{18}{1}$ *facit* $\frac{18}{1}$.

But if you would assign it any other denominator, multiply the whole number by the denominator assign'd, and place the product for a numerator over the assign'd denominator.

EXAMPLES.

Reduce 12 to a fraction, whose denominator let be 8. $\frac{12 \times 8}{8}$ *facit* $\frac{96}{8}$.

Or if 6 were to be made a fraction, and its denominator to be 7, it would become $\frac{42}{7}$.

For the reason of this rule consider the two foregoing notes.

Second, To reduce a mix'd fraction into an improper one,

Multiply the whole number by the denominator of the fraction given, and take in the numerator; the product place for a new numerator over the denominator given.

EXAMPLES.

Reduce $5\frac{3}{4}$ to an improper fraction, *facit* $19\frac{3}{4}$

$$\begin{array}{r} 3 \\ \hline 16 \\ \hline 3 \end{array}$$

So $12\frac{1}{2}$ reduc'd to an improper fraction will be $24\frac{1}{2}$.

The reason of this rule is the same as the foregoing, there being no difference in the operation, but the taking in the given *numerator*.

Third, To reduce an improper fraction into its equivalent whole or mix'd number.

R U L E.

Divide the *numerator* by the *denominator*, the quotient gives the whole number contain'd : But if any thing remains, (as in the ad example) it must be plac'd as a new *numerator* over the given *denominator*.

EXAMPLES.

Reduce $3\frac{1}{2}$ to its equivalent whole number.

$$8)32(4 \text{ facit.}$$

$$\begin{array}{r} 0 \\ 6)45(7\frac{1}{2} \text{ facit.} \\ \underline{42} \\ 3 \end{array}$$

This being only the reverse of the foregoing rule, the same reason still holds,

Fourth,

Fourth, To reduce a compound fraction to a single one, *i. e.* a fraction of a fraction, to a fraction of an unit.

R U L E.

Multiply all the *numerators* together for a *numerator*, and all the *denominators* for a *denominator*.

E X A M P L E S.

Reduce $\frac{3}{4}$ of $\frac{1}{2}$ to a single fraction, *facit* $\frac{3}{8}$.

Reduce $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ to a single fraction.

$\begin{array}{r} 4 \\ \hline 6 \\ \hline 24 \\ \hline 7 \\ \hline 168 \end{array}$	$\begin{array}{r} 5 \\ \hline 8 \\ \hline 40 \\ \hline 9 \\ \hline 360 \end{array}$	$\frac{168}{360}$ <i>facit</i> :
---	---	----------------------------------

So also $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{1}{2}$.

The reason of this operation will best appear by representing the unit or whole number by a line, which, according to the first example, must be supposed to be divided into 4 parts, and each of these parts again into 3 smaller parts; thus,



Then, as $\frac{1}{4}$ of the unit will denote 1 of the larger divisions, so $\frac{2}{3}$ of that fourth must signify only 2 of the lesser divisions; consequently if I would express what part of the whole line $\frac{2}{3}$ of $\frac{1}{4}$ is, it will plainly appear to be $\frac{1}{6}$; or, had the compound fraction been $\frac{2}{3}$ of $\frac{1}{4}$, the single fraction equal to it had been $\frac{1}{6}$, &c.

Fifth, To reduce a fraction into its lowest terms.

R U L E.

Divide the *numerator* and *denominator* by any figure, so that nothing may remain, and the quotients will be a new fraction of the same value with that given.

The rule generally given for finding the greatest common measure, or number, to divide your given fraction by, is this,

Divide the *denominator* by the *numerator*, and the *numerator* by the *remainder*, if any. So continuing to make the last divisor the dividend, and the remainder the divisor, till nothing remains, the last divisor will be the greatest common measure, or number, by which you can divide your fraction.

E X A M P L E.

What is the greatest common measure by which $\frac{208}{684}$ can be divided ?

$$\begin{array}{r}
 208 \overline{) 684} (3 \\
 \underline{624} \\
 60 \\
 60 \overline{) 208} (3 \\
 \underline{180} \\
 28 \\
 28 \overline{) 60} (2 \\
 \underline{56} \\
 4 \\
 \text{Answer, } 4 \overline{) 28} (7 \\
 \underline{28} \\
 0
 \end{array}$$

$\frac{208}{684}$ then being divided by 4, gives $\frac{52}{171}$, which are its lowest terms.

But this way of finding the common measure, is too tedious, often making more work than it saves. Observe therefore these more practical directions.

If you cannot at once discover the greatest number you may divide by, if both your *numerator* and *denominator* are even numbers, you may always halve them, and often that way reduce your fraction to advantage.

The

Thus the fraction given in the foregoing example, divided twice by 2, gives, as before, $\frac{1}{2}$.

$$2 \frac{228}{84} | \frac{114}{42} | \frac{57}{21},$$

and $\frac{1}{2}$ may be reduc'd, by continual halving, to $\frac{1}{4}$,

$$2 \frac{228}{84} | \frac{114}{42} | \frac{57}{21} | \frac{28}{21} | \frac{14}{10} | \frac{7}{5}.$$

Also, when both your *numerator* and *denominator* have cyphers on the right hand, you may abbreviate the fraction, by striking off an equal number from both;

$$\text{Thus, } \frac{41}{88} \text{ or, } \frac{41}{88} :$$

Or, if the right hand figures of your *numerator* and *denominator*, are both fives, or one a five and the other a cypher (as the remainder of the last example) you may always divide them by 5,

$$\text{thus, } \frac{41}{88}, \text{ divided by } 5 \\ \text{is } \frac{8}{17} \text{ equal to } \frac{41}{88}.$$

$$\text{So also } \frac{14}{28} \text{ reduc'd to } \frac{1}{2}.$$

Thus, In the Rule of Three, we abbreviate our statings by the same rule, and on the same reason; for the second and third numbers multiply'd together, are as a *numerator* to the first number, their *denominator*.

For the reason of this rule, see the 2d note.

synth. To reduce fractions of divers denominations, into one of the same value.

R U L E.

Multiply each of the given *numerators* into all the given *denominators*, except its own, and the several products

products will be so many new *numerators*, whose common *denominator* must be the product of all the given *denominators* multiply'd together.

E X A M P L E S.

Reduce $\frac{3}{8}$, $\frac{5}{4}$, and $\frac{6}{4}$, into one denomination.

To do which,

First, The *numerator* 3, must be multiply'd by the *denominators* 8 and 10, which will produce 240

Secondly, the *numerator* 5, multiply'd by the *denominators* 4 and 10, will produce 200

Thirdly, The *numerator* 6, multiply'd by the *denominators* 4 and 8, will produce 192

Then the three *denominators* 4, 8, 10, multiply'd together, will give 320 for a common *denominator*.

See the work.

$\frac{3}{8}$	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{4}{8}$
24	20	24	32
10	10	8	10
240	200	192	320
1st Numer.	2d Numer.	3d Numer.	Denominator.

$$\frac{348}{320} \quad \frac{500}{320} \quad \frac{192}{320}$$

So $\frac{3}{8}$, $\frac{5}{4}$, $\frac{6}{4}$, and $\frac{4}{8}$, reduc'd into one denomination, will be $\frac{348}{320}$, $\frac{500}{320}$, $\frac{192}{320}$, $\frac{400}{320}$.

See

See the work.

$\frac{5}{8}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{9}{10}$
$\frac{40}{12}$	$\frac{30}{12}$	$\frac{40}{8}$	$\frac{90}{8}$
$\frac{480}{15}$	$\frac{360}{15}$	$\frac{320}{15}$	$\frac{720}{12}$
$\frac{2400}{480}$	$\frac{1800}{360}$	$\frac{1600}{320}$	$\frac{8640}{8}$
7200	5400	4800	
1st Numer.	2d Numer.	3d Numer.	4th Numer.

$$\begin{array}{r}
 10 \\
 8 \\
 \hline
 80 \\
 12 \\
 \hline
 560 \\
 15 \\
 \hline
 4800 \\
 960 \\
 \hline
 14400 \\
 \text{Denominator.}
 \end{array}$$

The foregoing is the common rule for reducing fractions of several denominations into one: it may also be done by first finding the common denominator as before directed, and then to produce the new numerators dividing the said common denominator severally by each of the given denominators, and multiplying the quotients by the respective numerators; the products will be the several new numerators.

But,

But, as by abbreviation, fractions are often reduc'd into much lower terms than they are at first given in; so here, a common *denominator* may sometimes be found much smaller than that arising by the preceding method. The rule therefore for discovering the least common *denominator* to fractions of different denominations, is as follows, *viz.*

R U L E.

Having first by the fifth sort of reduction, reduc'd all your given fractions to their lowest terms, cancel or strike out all such of the *denominators* as are aliquot parts of others, then drawing a line under all the *denominators*, if two or more can be divided by any such number, so that nothing may remain, divide by such number; and place the quotients under the line, and with them, in their proper places, all such of the *denominators* as would not admit of such division. If any of the said numbers or *denominators* so brought down, are capable of further abbreviation, draw another line, and proceed as before, so continuing as low as possible; then multiply continually, one into another, the several divisors, and the quotients arising, or numbers remaining under the last line, and their product will be the least common *denominator*. The new *numerators* to which may be found, as before, by dividing the said common *denominator*, by each given *denominator*, and multiplying the quotients by their respective *numerators*.

E X A M P L E.

Reduce $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ into one denomination.

$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

4 5

Answer, $\frac{10}{40}, \frac{5}{40}, \frac{1}{40}$

The common *denominator* 40 is thus produc'd.

First, The *denominator* 4, being an aliquot part of 8, is cancell'd, then the *denominators* 8 and 10, are divided by 2. Lastly, The divisor $2 \times 4 \times 5 = 40$.

The

The reason of this sixth sort of reduction, is evident from the 2d of the preceding notes ; for as both the *numerator* and *denominator* of each given fraction are equally multiply'd by all the other *denominators*, consequently the new fractions thence arising, must be equal to the fractions given.

Seventh, To value a fraction, or reduce it to the known parts of an integer.

R U L E.

Multiply the *numerator* by the next inferior denomination, and divide the product by the *denominator*, the quotient shews the parts sought, and the remainder becomes a new *numerator* to the given *denominator* ; which must still be valued by the same rule, proceeding till you have brought it into the least known parts of the integer.

E X A M P L E S.

1st. What's the value of $\frac{3}{4}$ of a pound sterling ?

$$\begin{array}{r} 3 \\ 20 \\ \hline 5) 60 \\ 12 \text{ Shillings.} \end{array}$$

2^d. What's

[150]

2d. What's the value of $\frac{1}{4}$ of a pound Sterling?

$$\begin{array}{r}
 13 \\
 20 \\
 \hline
 49) 260 \text{ (5 } s. \text{ d. qua.} \\
 245 \\
 \hline
 15 \\
 12 \\
 \hline
 49) 180 \text{ (3} \\
 147 \\
 \hline
 33 \\
 4 \\
 \hline
 49) 132 \text{ (2} \\
 98 \\
 \hline
 34
 \end{array}$$

3d. What hundreds, quarters, and pounds, are contain'd in $\frac{1}{4}$ of a ton?

$$\begin{array}{r}
 53 \\
 20 \\
 \hline
 94) 1060 \text{ (11 } h. \text{ q. r. lb.} \\
 94 \\
 \hline
 120 \\
 94 \\
 \hline
 26 \\
 4 \\
 \hline
 94) 104 \text{ (1} \\
 94 \\
 \hline
 10 \\
 28 \\
 \hline
 94) 280 \text{ (2} \\
 188 \\
 \hline
 92
 \end{array}$$

There

There will be no difficulty in accounting for this rule, if we consider but the particular working of any one example: Thus in the 1st, $\frac{1}{4}$ of a pound Sterling are given to be valued: Now as 20s. make a pound, so consequently any part of a pound must be 20 times as great a part of a shilling; therefore $\frac{1}{4}$ of a pound make $\frac{20}{4}$ of a shilling; which being an improper fraction, its *numerator* is divided by its *denominator*, to find the units or whole numbers (which in this case must be shillings) contain'd in it; according to the directions of the 3d sort of *Reduction*.

By this rule are *remainders* of farthings in the Rule of Three, &c. valued. See page 26.

ADDITION of FRACTIONS.

WHEN the given fractions are of one denomination, all you have to do, is, to add their *numerators* together, and to place the sum for a new *numerator*, over the *denominator* given; thus, $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{1}{6}$, added together, make $\frac{4}{6}$, or $1\frac{2}{3}$.

But when amongst the given sums, there are either compound fractions, or single ones of different denominations, they must be prepar'd by reduction, before they can be added.

EXAMPLES.

1st. What's the sum of $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{1}{6}$?

According to the 6th sort of reduction, the given fractions being brought into one denomination, they will be $\frac{1}{6}$, $\frac{2}{6}$, $\frac{1}{6}$, the *numerators* of which being added, the sum is $\frac{4}{6}$, or $1\frac{2}{3}$.

2d. Add

2d. Add $\frac{1}{4}$ of $\frac{1}{8}$, and $\frac{1}{8}$ of $\frac{1}{4}$ and $\frac{1}{8}$ together.

Reduce the compound fractions to single ones, by the 4th rule of reduction; and instead of $\frac{1}{4}$ of $\frac{1}{8}$, and $\frac{1}{8}$ of $\frac{1}{4}$, you will have $\frac{1}{32}$ and $\frac{1}{32}$, to add to your $\frac{1}{8}$; all which single fractions being reduc'd, as before, to one denomination, they will be $\frac{1}{96}$, $\frac{1}{96}$, and $\frac{1}{96}$; which added, makes $\frac{3}{96}$, or $\frac{1}{32}$.

3d. Add $4\frac{1}{2}$ and $3\frac{1}{2}$ and $\frac{1}{2}$ together.

When mix'd numbers are given to be added, reduce only their fractional parts to one denomination, and add their sum to the total of the whole numbers.

$$\begin{array}{r}
 44 \quad 14 \quad 14 \quad 4 \text{ --- } 42 \\
 3 \text{ --- } 14 \\
 \text{---} \quad \quad \quad 24 \\
 \text{---} \quad \quad \quad 80 \\
 \text{fact 7 ---} \\
 84
 \end{array}$$

More EXAMPLES.

Add $8\frac{1}{2}$ and $3\frac{1}{2}$ and $\frac{1}{2}$ together.
Fact 14 $\frac{1}{2}$.

What's the sum of $\frac{1}{4}$ of $\frac{1}{8}$, and $4\frac{1}{2}$ and $\frac{1}{2}$?
Answer, $5\frac{1}{8}$.

SUBTRACTION of FRACTIONS.

Subtraction of Fractions is also nothing (after the given fractions are prepar'd by reduction) but taking one numerator from the other.

EXAMPLES.

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or if the
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EXAMPLES.

1st. From $\frac{1}{2}$ take $\frac{1}{3}$ 27
16

prepar'd $\frac{27}{48}$ $\frac{16}{48}$ $\frac{11}{48}$ Remainder.

2d. From $5 \frac{2}{3}$ take $1 \frac{1}{3}$.

First, The mix'd number $5 \frac{2}{3}$ being reduc'd to the improper fraction $\frac{17}{3}$, by the second rule of reduction, proceed as before.

From $\frac{17}{3}$ take $1 \frac{1}{3}$, or prepared.

From $\frac{17}{3}$ take $\frac{4}{3}$ Rem. $\frac{13}{3}$ or $4 \frac{1}{3}$.

Or without reducing the mix'd number to an improper fraction, make only the fractional parts of one denomination, and you will then have $\frac{2}{3}$ to take from $\frac{1}{3}$; which, because you cannot do, you must borrow an unit, *i. e.* $\frac{1}{3}$, then $\frac{2}{3}$ from $\frac{1}{3}$, there will remain $\frac{1}{3}$; to which adding the $\frac{1}{3}$, the remainder will then be $\frac{2}{3}$; and the unit borrowed being deducted from the whole number 5, there will remain in all $4 \frac{2}{3}$, as before.

More EXAMPLES.

From $5 \text{ l. } \frac{2}{3}$ take $1 \frac{1}{3}$ of $\frac{1}{3}$ Rem. $5 \text{ l. } \frac{1}{3}$.

From $\frac{12}{10}$ take $\frac{1}{5}$ of $\frac{2}{5}$. Rem. $\frac{11}{10}$.

MULTIPLICATION of FRACTIONS.

IF the numbers given, are whole, or mix'd, they must first be brought into improper fractions; or if they are compound fractions, they must be reduc'd to single ones; but if the numbers given, are both

both either proper or improper single fractions, you have only to multiply the two given *numerators* together for a new *numerator*, and the given *denominators* for a *denominator*.

E X A M P L E S.

- 1st. Multiply $\frac{3}{8}$ by $\frac{1}{2}$ *facit* $\frac{3}{16}$, or by abbreviation $\frac{3}{16}$.
- 2^d. Multiply $\frac{3}{8}$ by $\frac{1}{2}$ *facit* $\frac{3}{16}$ or $\frac{3}{16}$.
- 3^d. Multiply 4 by $\frac{1}{2}$. 4 by the first sort of reduction, reduc'd to a fraction, is $\frac{4}{1}$, which multiply'd by $\frac{1}{2}$, makes $\frac{4}{2}$, or 2 .
- 4th. Multiply $7\frac{1}{2}$ by $8\frac{1}{2}$. These mix'd fractions, reduc'd by the 2^d rule of reduction to improper ones, make $\frac{15}{2}$ and $\frac{17}{2}$, which multiply'd together, make $\frac{255}{4}$, or $63\frac{3}{4}$.

Two things require explanation in multiplication of fractions, 1st. Why, when the multiplier is a proper fraction, the product is always less than the multiplicand?

2^d. Why the *denominators* are multiply'd as well as the *numerators*, whereas in addition, we only find the sum of the *numerators*?

Now the reason why a number multiply'd by a fraction is decreas'd, will appear by considering that as an unit is no multiplier, *i. e.* that any number multiply'd by 1, remains still the same, as once 4 is 4, &c. so to multiply any thing by a fraction, which is but a part of 1, must consequently produce but such a proportionable part of the multiplicand, as such fraction is of an unit. Thus 4 multiply'd by $\frac{1}{2}$, produces but 2, the half of 4; which 4 being made a fraction by the 1st rule of reduction, is $\frac{4}{1}$; so that the numbers resolve themselves into a compound fraction; and the product of 4, multiply'd

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ply'd by $\frac{1}{2}$, is plainly the $\frac{1}{2}$ of $\frac{1}{2}$; which by the rule for reducing a compound fraction, makes $\frac{1}{4}$, or $\frac{1}{2}$; the reason therefore of the work is clear, from the explanation of the 4th sort of reduction.

More *EXAMPLES.*

Multiply $9\frac{1}{2}$ by $\frac{2}{3}$, *facit* $3\frac{1}{3}$.

Multiply $3d. \frac{1}{2}$ by $3d. \frac{1}{4}$, *facit* $14d. 1\frac{1}{8}$.

DIVISION of FRACTIONS.

WHEN you have made the same preparation of your numbers as directed in multiplication, multiply the *denominator* of your divisor by the *numerator* of your dividend, for a *numerator*; and the *denominator* of your dividend, by the *numerator* of your divisor, for a *denominator*.

EXAMPLES.

1st. Divide $\frac{2}{3}$ by $\frac{1}{4}$, *facit* $\frac{8}{3}$, or $2\frac{2}{3}$.

2^d. Divide 7 by $\frac{1}{4}$, prepar'd by the first rule of reduction, it will be $\frac{7}{1}$ by $\frac{1}{4}$, *facit* $\frac{28}{1}$, or $9\frac{1}{4}$.

3^d. Divide $4\frac{1}{2}$ by $2\frac{1}{3}$, reduc'd by the 2d rule, it is $\frac{9}{2}$ by $\frac{2}{3}$, which, divided, gives $\frac{27}{4}$, or $1\frac{3}{4}$.

4th. Divide $\frac{2}{3}$ of $\frac{1}{4}$ by $\frac{1}{2}$ of $\frac{1}{3}$, reduc'd by the 4th rule, $\frac{2}{3}$ by $\frac{1}{6}$, *facit* $\frac{4}{1}$.

There is nothing in the whole practice of fractions, that more requires an explanation, than the manner of performing division, so different from that of whole numbers. Now, as the effect of division is the finding how often one number is contain'd in another; or, which is the same thing, what part of the dividend the divisor is; so it is plain,

plain, that if the divisor is unity or 1, the quotient must be equal to the dividend, consequently, in what proportion soever the divisor exceeds unity, in such proportion must the quotient be less than the dividend, and in what proportion soever the divisor is less than unity, by the same proportion must the quotient exceed the dividend. Thus, suppose the number 4 was given for a dividend, if 1 is propos'd for its divisor, the quotient will be also 4; but if we make the divisor 2, twice the former divisor, the quotient will be but 2, half the former quotient. Again, let the divisor be $\frac{1}{2}$, i. e. half of 1, and the quotient will then be found to be 8, or 8, twice as much as the first quotient, as the divisor is here but half the first divisor. From whence appears the reason,

1st. Why dividing a number by a proper fraction gives a quotient greater than the dividend, as multiplying a number by a fraction was before prov'd to give a product less than the multiplicand.

2^d. Why the operation is to be perform'd in the manner directed; for, as the greater the denominator is, the less is the value of the fraction, and consequently that the quotient must increase in the same proportion, is a clear reason for multiplying the numerator of the dividend by the denominator of the divisor, for the numerator of the quotient; so in like manner, as the increase of the numerator of a fraction is the increase of the value of such fraction, it is therefore as plain, the value of the quotient must be diminished in the same proportion, by the increase of its denominator, which is certainly effected by multiplying the denominator of the dividend, by the numerator of the divisor. Thus $\frac{1}{2}$ divided by $\frac{1}{4}$, gives $\frac{4}{2}$; but divided by $\frac{1}{8}$, gives but $\frac{8}{2}$, the value of the quotient still decreasing in the same proportion with the increase of the value of the divisor.

More *EXAMPLES*.

Divide $\frac{2}{3}$ by $\frac{1}{3}$. *Quot.* $2 \frac{2}{3}$.

Divide $9 \frac{1}{2}$ by $\frac{2}{3}$ of $\frac{1}{4}$. *Quot.* $31 \frac{3}{4}$.

The DIRECT RULE of THREE
IN
VULGAR FRACTIONS.

After the numbers are prepared by reduction, state and work the question by the rules given whole numbers.

EXAMPLES.

Q. What will $\frac{1}{2}$ a pound of snuff cost, if 122 lb $\frac{1}{4}$ the same come to 7 l. $\frac{1}{3}$?

lb. l. lb.

If 122 $\frac{1}{4}$ cost 7 $\frac{1}{3}$, what will $\frac{1}{2}$?

Prepar'd by the 2d rule of reduction, it will stand

If $244 \frac{1}{2} \dots \frac{29}{3} \dots \frac{1}{2}$?

In the second and third numbers multiply'd together, produce $\frac{29}{6}$, which divided by $\frac{29}{3}$, gives $\frac{1}{2}$; which, valu'd by the 7th rule of reduction, is 3 d. 2 qn. $\frac{1}{3} \frac{2}{3}$.

H

2d. Bought

2d. Bought 87 gallons and an $\frac{1}{2}$ of brandy for 40
at the same rate, what would $\frac{1}{2}$ a gallon cost?

Gall.	l.	Gall.
If 87 $\frac{1}{2}$ cost 40, what will $\frac{1}{2}$?		
prepar'd 17 $\frac{1}{2}$	$\frac{40}{1}$	$\frac{1}{2}$
then $\frac{40}{1} \times \frac{1}{2} = \frac{40}{2} = 20$		

l. s. d. q.

or 4 6 3

Another R U L E.

When your question is stated, and your number prepar'd, multiply the *denominator* of the first number by the *numerators* of the second and third, and place the product for a *numerator*: Then multiply the *numerator* of the first number by the *denominator* of the second and third, and place the product for its *denominator*; and the new fraction so found, is the answer of the question. Take the stating of the question for an

E X A M P L E.

If $17\frac{1}{2} \dots \frac{40}{1} \dots \frac{1}{2}$ facit 20 .

For 2, the *denominator* of the first number, multiply'd by 40 and 1, the *numerators* of the second and third numbers, is 80.

And 175, the *numerator* of the first, multiply'd by 1 and 2, the *denominators* of the second and third, is 350. These products plac'd fractionally, make $350/80$, as before.

3d. What will 82 C. of Sugar come to, if $\frac{1}{2}$ of a C. costs $1\frac{2}{3}$ of a pound Sterling?

C.	l.	C.
If $\frac{1}{2}$ of $\frac{1}{3}$ cost $1\frac{2}{3}$, what will 82?		
$3\frac{2}{3}$	$1\frac{2}{3}$	82
Facit $236\frac{1}{3}$ or 262 l. 8 s.		

More *E X A M P L E S.*

What will $\frac{1}{4}$ of an ounce of snuff cost, if the lb comes to 8s. $\frac{1}{5}$?

Answer, 1 d. 2 qua. $\frac{1}{4}$.

At 1 d. $\frac{1}{2}$ per ounce, what will 5 C. $\frac{1}{2}$ come to?

Answer, 61 l. 12 s.

The INDIRECT RULE of THREE
IN
VULGAR FRACTIONS.

Here also you have only to observe the directions given in whole numbers:

Or this R U L E.

After the numbers are stated and prepared, multiply the *numerators* of the first and second numbers, by the *denominator* of the third, for a *numerator*; and the *denominators* of the first and second, by the *numerator* of the third number, for a *denominator*; and the fraction so found, is the *Answer*. See both ways in the example.

E X A M P L E.

How many yards of stuff, $\frac{1}{4}$ yard wide, are equal 36 $\frac{1}{4}$ yards of $\frac{1}{4}$ wide?

If $\frac{1}{4}$ ——— 36 $\frac{1}{4}$ ——— $\frac{1}{4}$

Prepar'd $\frac{1}{4}$ $\frac{141}{4}$ $\frac{1}{4}$

Then $\frac{1}{4} \times \frac{141}{4} = \frac{141}{16} \cdot \frac{141}{16} \div \frac{1}{4} = \frac{270}{8}$ or 54 $\frac{1}{8}$.

H 2

So

So also 3, 145, and 2, the *numerators* of the first and second numbers, and the *denominator* of the third, multiply'd together, is 870.

And 4, 4, and 1, the *denominators* of the first and second, and *numerator* of the third, multiply together, produce 16

These numbers fractionally plac'd, make $\frac{870}{16}$, before.

More EXAMPLES.

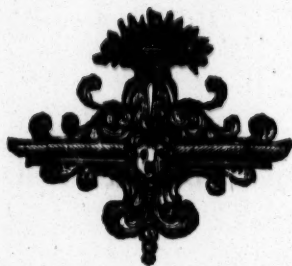
If *A.* lends *B.* 25 *l.* $\frac{1}{2}$ for 6 months $\frac{1}{2}$, how long may he keep 10 *l.* $\frac{1}{4}$ of *B.*'s, to requite himself?

Answer, 16 months $2\frac{16}{10}$.

If *A.* keeps 100 *l.* $\frac{1}{8}$ of *B.*'s 4 months $\frac{1}{2}$, what sum must *A.* lend *B.* for 2 years $\frac{1}{2}$ to requite him?

Answer, 14 *l.* $\frac{3}{4}\frac{1}{2}$.

The End of the First Part.

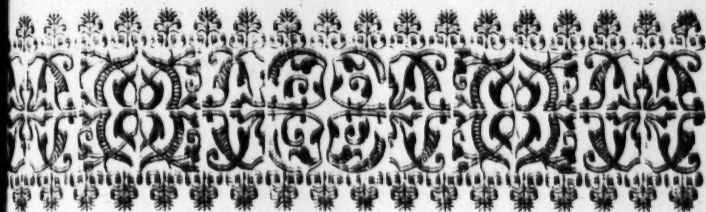


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THE
SECOND PART:
TREATING OF
Decimal Arithmetick.

THIS part of Arithmetick is very compendious, and therefore very useful, especially in calculations of Interest, valuing annuities, &c. But by reason some fractions of coin, weight, and measure, cannot be exactly express'd by decimal parts, but that the numbers will often circulate, is not always convenient to work with them. Mr. Cunn hath, indeed, in his late treatise on Fractions, very curiously shewn how they may be reduced with least loss; but since such exactness deploys their brevity, I cannot but think, in such cases, vulgar fractions preferable.



CHAP. I.

Of NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION of DECIMALS.

NUMERATION.

IN DECIMAL FRACTIONS, the *integer* or whole thing, (whether it be time, coin, weight, or measure, as one year, one pound Sterling, one pound weight, &c.) is suppos'd to be divided into tenths, or ten equal parts; and those parts again into tenths, and so on *ad infinitum*.

So that the *denominator* of a decimal fraction being always known to consist of an unit, with as many cyphers as the *numerator* hath places, is seldom set down; the parts being only distinguish'd from whole numbers, by a point or comma prefix'd; thus, 1 stands for 10, 25 for 100, 123 for 1000, &c.

But the different value of the several places, will more plainly appear from the following table.

TABLE.

Whole Numb.						Decimal Parts.						
6	5	4	3	2	1,	2	3	4	5	6	7	
Units.						Parts of 1000000						
Tens.						Parts of 100000						
Hundreds.						Parts of 10000						
Thousands.						Parts of 1000						
X Thousands.						Parts of 100						
C Thousands.						Tenths, or Parts of 10						

Dr.

from whence it is evident,

1st. That as whole numbers increase by a tenfold proportion towards the left hand, from the unit's place; so decimal parts decrease, in the same proportion, towards the right.

2^d. That therefore cyphers, plac'd before decimal parts, decrease their value, by removing them farther from the comma, or unit's place.

Thus, $\begin{cases} 5, & \text{is } 5 \text{ parts of ten, or } \frac{1}{2}. \\ .05, & \text{is } 5 \text{ parts of an hundred, or } \frac{1}{20}. \\ .005, & \text{is } 5 \text{ parts of a thousand, or } \frac{1}{200}. \end{cases}$

3^d. But that cyphers, after decimal parts, alter not their value, because they do not change their places.

For 5, 50, 500, &c. are each but $\frac{1}{2}$ of an unit, or 1; the 5 still keeping its first place.

These things understood, the rest is easy: But let the reader be sure he does understand them before he proceeds.

ADDITION.

HAVING set down your proposed numbers, units under units, tens under tens, and parts under parts of like value, add them as if they were all whole numbers, separating so many decimal parts in the total, as were in any of the given numbers.

EXAMPLES.

Add 5,42, and 38,583, and 421,06, and ,834 together.

*These numbers,
rightly plac'd,
will stand thus :*

$$\begin{array}{r} 5,42 \\ 38,583 \\ 421,06 \\ ,8345 \\ \hline \end{array}$$

Total, 465,8975

What's the sum of 4,06, and 21,734, and 523,864?

$$\begin{array}{r} 4,06 \\ 21,734 \\ 523,864 \\ \hline \end{array}$$

Answer, 549,6582

SUBTRACTION.

HAVING plac'd the numbers as directed in addition, work still as if they were all whole numbers.

EXAMPLES.

From 829,38275 take 10,49561

$$\begin{array}{r} 829,38275 \\ 10,49561 \\ \hline \end{array}$$

828,88714 Remainder.

Take 23,279 from 451,15842

$$\begin{array}{r} 451,15842 \\ 23,279 \\ \hline \end{array}$$

Remains 427,87942

MULTI-

MULTIPLICATION.

Here the numbers are to be set down, (without regard to the value of their places) and work'd in all respects the same as in whole numbers; only observe to separate so many places of decimal parts on the right hand of the product, as were in both multiplicand and multiplier.

EXAMPLES.

Multiply 2,735641
by 54382

$$\begin{array}{r}
 21 \quad | \quad 5471282 \\
 82 \quad | \quad 885128 \\
 1094 \quad | \quad 06923 \\
 13678 \quad | \quad 2564 \\
 \quad \quad | \quad 205 \\
 \hline
 1,4876 \quad | \quad 5628862
 \end{array}$$

Multiply 73,82564365
by 8,7434592

$$\begin{array}{r}
 14765128730 \\
 66443079285 \\
 3 \quad 6912821825 \\
 29 \quad 530257460 \\
 221 \quad 47693095 \\
 2953 \quad 0257460 \\
 51677 \quad 950555 \\
 590605 \quad 14920 \\
 \hline
 \hline
 \end{array}$$

Product 645,491 503167514080

A compendious way to multiply numbers that have many decimal places; by omitting all the figures to the right of the perpendicular lines in the examples above.

R U L E.

Place one of the numbers just as it is given, for a multiplicand, and under that decimal place which you would have be the last in the product, set the unit's place of your other number; then reversing all the other figures, multiply with them in their order, beginning always with that figure of the multiplier, which stands over the number you work with; having also respect to the increase you think

H 5

would

would have been brought from the omitted figures on the right hand, had they been multiply'd.

Note, the usual allowance for such omitted figures is as follows, *viz.* If the next figure on the right hand of that you begin with in the multiplicand, multiply'd into the figure of the multiplier you are working with, gives a product betwixt 5 and 15, carry 1; if the product be above 15, and less than 25, carry 2; and if it arise to any number betwixt 25 and 35, carry 3, &c.

To prove the certainty of the rule, we will take the same examples before given.

EX A M P L E S.

Multiply 2,735641 by ,54382, and produce only four places of decimals.

$$\begin{array}{r} 2,735641 \\ 28345,0 \\ \hline \end{array}$$

13678 the Increase here carried is 3 for 5×6
 1094 the Increase here is 2 for 4×5
 82 the Increase here is 1 for 3×3
 22 the Increase here is 6 for 8×7

$$\begin{array}{r} 1,4876 \\ \hline \end{array}$$

Note, There being no units in the multiplier, its place is supply'd with a cypher.

Multiply 73,82564365 by 8,7434592, and produce but 3 places of decimals.

$$\begin{array}{r} 73,82564365 \\ 2954347,8 \\ \hline \end{array}$$

$$\begin{array}{r} 590605 \\ 51678 \\ 2953 \\ 222 \\ 29 \\ \hline \end{array}$$

4 the Increase carried for 5×7

$$\begin{array}{r} 645,491 \\ \hline \end{array}$$

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Note, To multiply any decimal number by 10, 100, 1000, &c. is only to remove the mark of separation so many places towards the right hand as there are cyphers.

Thus 8,3564537 $\left\{ \begin{array}{l} 10, \\ 100, \\ 1000, \\ 10000, \end{array} \right\}$ multiply'd by $\left\{ \begin{array}{l} 83,564537 \\ 835,64537 \\ 8356,4537 \\ 83564,537, \&c. \end{array} \right\}$ is

D I V I S I O N.

THIS rule also being work'd the same as in whole numbers, the only difficulty is to value the quotient; which may be done by either of the following general rules, *viz.*

R U L E 1.

The first figure in the quotient, is always of the same value with that figure of the dividend, which answers, or stands over the place of units in the divisor. Or,

R U L E 2.

The quotient must always have so many decimal places, as the dividend has *more* than the divisor.

Note, 1st. If the divisor and dividend have both the same number of decimal parts, (as in the second example) the quotient will be all a whole number.

2^d. If the dividend hath not so many places of decimals as are in the divisor, (as in example the 3^d) so many cyphers must be annex'd to the dividend, as will make them equal; and the quotient will then, as before, be all a whole number.

3^d. But if when division is done, the quotient has not so many figures as it should have places of decimal parts, (as in the last example) so many cyphers must be prefix'd, as there are places wanting.

EXAMPLES.

17. Divide 10,55439 by 4,569

$$4,569 \overline{) 10,55439} (2,31$$

$$\underline{14163}$$

$$\underline{4569}$$

...

Here, according to the first rule, 4, the unit's place of the divisor, standing under units, or the first place of whole numbers in the dividend; 2, the first figure placed in the quotient, must also be made the first place of whole numbers, by placing the point of separation immediately after it.

Or, according to the second rule, there being five places of decimals in the dividend, and but three the divisor, two places must be separated off in the quotient, to make up the number.

EN. 2d. Divide 85643,825 by 6,321

$$6,321 \overline{) 85643,825} (13549$$

$$\underline{22433}$$

$$\underline{34708}$$

$$\underline{31032}$$

$$\underline{57485}$$

$$\cdot 596$$

Here 6, the place of units in the divisor, standing under 8, the fifth place of whole numbers in the

the dividend 1, the first figure of the quotient, must also stand in the same place, which makes all the quotient whole numbers.

And so it must also be by the second rule, because the dividend has no more places of decimals than the divisor.

Ex. 3. Divide 7382,54 by 6,4352.

$$6,4352 \overline{) 7382,5400} (1147$$

Here there not being so many places of decimals in the dividend, as in the divisor, two cyphers are added to make them even; and being so, the quotient is all a whole number, as before.

Ex. 4. Divide ,08516438 by 423

$$423 \overline{) ,08516438} (,00020133$$

$$\begin{array}{r} 20133 \\ \underline{846} \\ 1413 \\ \underline{846} \\ 5678 \\ \underline{423} \\ 1448 \\ \underline{423} \\ 179 \end{array}$$

In this example, the quotient consisting but of five figures, whereas there should, by the rules, be eight places of decimal parts, three cyphers are prefix'd to make up the number, as was directed in *Note the third.*

There

There is also a way of contracting the work of a large division, somewhat like that compendious method before taught of multiplying numbers of many places of decimals, and bringing out only so many in the product as are necessary for the answer required.

R U L E.

Having consider'd by the first rule given for valuing the quotient, the value of the first figure therein to be placed, and accordingly determin'd how many figures it is necessary it should consist of, you may from thence judge whether any, or how many of the right hand figures of your divisor may be neglected, so that reserving only so many figures of the dividend as are necessary for once answering the divisor, cut off the rest to the right hand as useless, and having set the proper figure in the first place of the quotient, work with it as usual; then omitting the right hand figure of your divisor, seek how often the other figures of your divisor are contain'd in the remainder, which figure being enter'd in the quotient, and work'd with as usual in division, (with regard, however, to the carriage that would have been brought from the omitted figures, as before directed in multiplication) thus continuing to neglect the right hand figure of your divisor every time you seek a new quotient figure, you will still be able to divide the remainder left by the last subtraction by the divisor so lessen'd, till you have brought out the determin'd number of figures in the quotient.

The following example, which is set down both at length, and the contracted way, will make all clear.

EXAMPLE.

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E X A M P L E.

3,4637528)95,4327|56463275(27,55183
69275056

261577004
242462696

191143086
173187640

179554463
173187640

63668232
34637528

290307047
277100224

132068235
103912584

28155651

3,46375|28)95,4327|56463275(27,5518
692750

261577
242463

19114
17319

1795
1732

63

35

28

26

2

And

And thus are all the figures on the right hand of the perpendicular line in the example work'd at length, sav'd by working after this contracted manner.

As multiplying by 10, 100, 1000, &c. is only removing the separating point of the multiplicand, so many places as there are cyphers in the multiplier, so, to divide by 10, 100, 1000, &c. is only removing the mark of separation, after the same manner, towards the left hand.

$$\begin{array}{l} \text{Thus } 83564.537 \\ \text{divided by} \end{array} \left\{ \begin{array}{l} 10, \\ 100, \\ 1000, \\ 10000, \end{array} \right\} = \left\{ \begin{array}{l} 8356.4537 \\ 835.64537 \\ 83.564537 \\ 8.3564537 \end{array} \right.$$

These examples being, I hope, sufficient to make division plain, we will now proceed to *Reduction of Decimals*.





C H A P. II.

Of REDUCTION of DECIMALS.

THERE are two sorts of *Reduction of Decimal Fractions*. 1st. changing a vulgar fraction to a decimal. 2. valuing a decimal by the known parts of an integer.

1st. To reduce or change a vulgar fraction to a decimal.

R U L E.

Place cyphers at pleasure after the *numerator*, and divide by the *denominator*.

E X A M P L E S.

Reduce $\frac{1}{4}$ to a decimal 4)1,00(,25 *facit*.

What decimal parts are equal to $\frac{1}{8}$?

8)3,000(,375 *facit*.

And so any part of coin, weight, measure, or time, &c. may be brought into a decimal, by first reducing it into a vulgar fraction, (which is done by placing the known parts of the integer as a denominator under the parts given) then annexing cyphers to the *numerator*, and dividing by the *denominator*, as before directed.

E X A M P L E S.

EXAMPLES.

What decimal parts of a pound are equal to 16s?
i. e. $\frac{1}{10}$ 20)16,0(,8 *fact.*

What's the decimal of 9 oz. the Inter a lb. Troy?
i. e. $\frac{9}{12}$ 12)9,00(,75 *fact.*

Note, If the given parts are of several denominations, they may be reduc'd either by so many distinct operations as there are different parts, or by first reducing them into their lowest denomination, and then one division will serve. See both ways in the

EXAMPLES.

Reduce 14s. 9d $\frac{1}{4}$. into decimal parts of a pound Sterling: By the first way there will be these three vulgar fractions to be chang'd, $\frac{1}{20}$, $\frac{9}{240}$, and $\frac{1}{480}$.

$\begin{array}{r} 20)14,00(,70 \\ \hline 960)3,0000(,0031 \\ \hline 1200 \\ \hline 240 \end{array}$	$\begin{array}{r} 240)9,0000(,0375 \\ \hline 1800 \\ \hline 1200 \\ \hline \cdot 00 \end{array}$	$\begin{array}{r} ,70 \\ ,0375 \\ ,0031 \\ \hline \text{fact } ,7406 \end{array}$
---	--	---

The other way. 14s. 9d $\frac{1}{4}$ being reduc'd to farthings.

$$\begin{array}{r} 12 \\ \hline 177 \\ \hline 4 \end{array}$$

will be 711

which divided by its denominator, } $960)711,0000(,7406$ is produc'd as (before.

$$\begin{array}{r} 3900 \\ \hline \cdot 6000 \\ \hline 240 \end{array}$$

What

What decimal parts of C. are equal to 3 grs. 18 lb ?

By the first way :

$$\begin{array}{r}
 \text{and } 1\frac{1}{2} \quad 4)3,00(.75 \qquad 112)18,0000(.1607 \\
 \underline{1,607} \qquad \qquad \qquad \underline{11,200} \\
 ,9107 \text{ fact.} \qquad \qquad \qquad \underline{6,800} \\
 \qquad \qquad \qquad \qquad \qquad \underline{1,600} \\
 \qquad \qquad \qquad \qquad \qquad \underline{5,200} \\
 \qquad \qquad \qquad \qquad \qquad \underline{1,600} \\
 \qquad \qquad \qquad \qquad \qquad 16
 \end{array}$$

By the second way :

$$\begin{array}{r}
 \text{grs. lb.} \\
 3 \quad 18 \\
 28 \\
 \hline
 102 \\
 \hline
 112 \text{ Denominator}
 \end{array}
 \qquad
 \begin{array}{r}
 112)102,0000(.9107 \\
 \underline{11,200} \\
 \underline{9,000} \\
 \underline{1,200} \\
 \underline{800} \\
 16
 \end{array}$$

There is also another way of reducing parts of different denominations, into decimals of the highest integer ; thus,

R U L E.

Bring the lowest parts into decimals of the next superior denomination ; and on the right hand of the decimal found, place the parts given of that said next superior denomination ; so proceeding, till you bring out the decimal parts of the highest integer requir'd, by still dividing the product by the next superior denominator ; for examples take those before given, viz.

Reduce

Reduce 14s. 9d $\frac{3}{4}$. into decimal parts of a pound Sterling.

$$\begin{array}{r} 4 \overline{) 3} \\ 12 \overline{) 9.75} \\ 20 \overline{) 14.8125} \\ \hline .7406 \end{array}$$

Here the 3 farthings being divided by 4, gives 75, to which the 9d. being prefix'd, it becomes 9.75; which divided again by 12, quotes .8125; before which the 14s. being also plac'd, it makes 14.8125; which being lastly divided by 20, the quotient .7406, is the answer.

What decimal parts of a C. are equal to 3 grs. 18 lb?

$$\begin{array}{r} 28 \overline{) 18} \\ 4 \overline{) 2.5714} \\ 4 \overline{) 1.6428} \\ \hline .9107 \end{array}$$

Note, Instead of dividing by 28, which would be troublesome, 'tis better to divide by 7 and 4, as in the example, 4 times 7 being 28.

Thus you see how easily the decimals, answering to any given parts of coin, weight, measure, &c. are found; and after the same manner are the following tables form'd.

But shillings, pence, and farthings, may more readily be reduc'd thus:

R U L E.

For the shillings, (if their number be even) set down their half in the first place of decimals, and let the second and third places be fill'd up with the farthings contain'd in the remaining pence and farthings, always remembring to add one when the number is, or exceeds 25; but if the number of shillings

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shillings be odd, the second place of decimals must also be increas'd by 5.

Thus the decimal of 8s. 5d $\frac{1}{2}$. will be ,422, 4 in the first place of decimals, standing for 8s. and 22 farthings being 5d $\frac{1}{2}$.

Again, 8s. 6d $\frac{1}{2}$ will be ,427.

1 being added to 26, the number of farthings in 6d $\frac{1}{2}$, because exceeding 25.

But, lastly, 9s. 6d $\frac{1}{2}$ will be thus express'd by decimals, viz. ,477.

5 being added to the second place, because the number of shillings is odd, the decimal of 1 shilling being ,05.

The second sort of reduction, is to value a decimal by the known parts of its integer; which is thus done :

R U L E.

Multiply the decimal given, by the known parts of the next inferior denomination, and ordering the product as directed in multiplication, still multiply the remaining decimals by the known parts of the next inferior denomination; thus proceeding 'till you have brought it into the least known parts of the integer, the figures standing on the left of the seperating points, will be the parts requir'd.

E X A M P L E S.

Ex. 1. What's the value of ,7691, the integer a pound Sterling, (i. e.) What shillings, pence, and farthings, are equal to ,7691 parts of a pound Sterling ?

	,7691	parts of a pound
multipl'y'd by	<u>20</u>	the shillings in 1 l.
produce shillings,	15,3820	and parts of a shilling,
which multiply'd by	<u>12</u>	the pence in 1 shilling,
make pence	4,5840	and parts of a penny,
which again multiply'd by	<u>4</u>	the farthings in 1 d.
give farthings	2,3364	and par. of a farthing.
		So

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So that the value of .7691 decimal parts of a pound Sterling, is 15 s. 4 d $\frac{1}{2}$.

Ex. 2. What ounces, pennyweights, and grains, are equal to .84362 decimal parts of a pound Troy?

Multiply'd by .84362 parts of a pound Troy,
12 the ounces in one pound,

Give ounces 10,12344 and parts of 1 ounce.
Multiply'd by 20 the pennyweights in 1 oz.

Give pennyweights 2,46880 and parts of 1 pennywt.
Multiply'd by 24 the grains in 1 pennywt.

187520
93760

Give grains 11,25120 and parts of 1 grain.

The Answer therefore is 10 oz. 2 dwts. 11 grs.

But the decimal parts of a pound Sterling, may be thus valu'd at sight :

R U L E.

The figure standing in the first place of decimals, doubled, gives shillings; but if the figure in the 2d place, is, or exceeds 5, one more must be added to their number; the second figure (if under 5) or its excess, (if above 5) join'd with the third, are so many farthings; only remember to abate 1 if their number amounts to 25; or 2, if near 50.

The reason of this abatement is, that as 1000 is the denominator of every decimal consisting of 3 places, so consequently by reckoning the figure standing in the said third place, as 10 many farthings, we thereby allow 1000 farthings to the pound, whereas indeed there are but 960, the over-
plus

plus therefore being 40 in 1000, is certainly 4 in a 100, or 1 in 25, &c. according to the direction above.

EXAMPLES.

Ex. 1. What's the value of ,375 parts of a pound Sterling?

Answer, 7 s. 6 d.

The 3 doubled is 6 shillings, to which 1 s. being added for the 5, in the second place, makes it 7 s. and 2 remaining, join'd with the 5 in the 3d place, being accounted 25 farthings, from which 1 being deducted, there remains 24 farthings, or 6 pence.

Ex. 2. What's the value of ,846 parts of a pound?

Answer, 16 s. 11 d.

The 8 doubled, making 16 s. and 2 abated from the 46 farthings, leaves 44 farthings, or 11 d.

Thus are shewn the ways both of finding the decimals answerable to any given parts of coin, weight, measure, &c. or the value of any given decimal in the known parts of its integer.

I have likewise annex'd TABLES ready calculated by the foregoing rules, answering to the same ends.

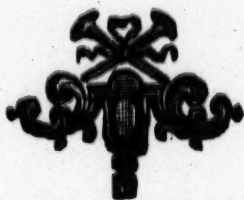
The use of which is very plain, one column in each table containing the number of shillings, ounces, drams, points, bushels, days, hours, or any thing else you may want. And against the said number, in the other column, the decimal parts answering. Thus against 3 pence in the first table, is found ,0125, and against 40 days in the fourth table, ,109589.

But if the number you want is not to be found in the said tables (many compound numbers having been omitted, to reduce them into so narrow a compass) you must add the parts of such two or three numbers together, as will compose it, and the total will be the decimal sought. Thus, if the decimal of 17 pennyweights, the integer of a pound Troy, were to be sought in the 2d table ,04166 the decimal of 10 pennyweights, and ,029166 the decimal of 7 pennyweights, being added together, the total ,070832 will be found the answer; or, if in table the fourth, you would find the decimal of 352 days,

$$\begin{array}{r} \text{add } ,821918 \\ ,136986 \\ ,005479 \\ \hline \end{array} \left. \begin{array}{l} \text{the} \\ \text{Decimals} \\ \text{of} \end{array} \right\} \begin{array}{r} 300 \\ 50 \\ 2 \end{array} \left. \begin{array}{l} \text{Days.} \end{array} \right\}$$

total ,964318 answering 352 Days.

and so of any other.



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Decimal Tables

Coin & Weight

Table 1 st				Table 2 ^d		Table 3 ^d	
Coin				Troy Weight		Apothecary's	
1ster: y ^e Integer				1 st the Integer		12 th y ^e Integer	
S	Dec:	S	Dec:	Ounces	Decim:	Quarters	Decim:
19	.95	9	.45	11	.916666	3	.75
18	.9	8	.4	10	.833333	2	.5
17	.85	7	.35	9	.75	1	.25
16	.8	6	.3	8	.666666	Pounds	Decim:
15	.75	5	.25	7	.583333	20	.1705371
14	.7	4	.2	6	.5	10	.0802680
13	.65	3	.15	5	.416666	9	.0401340
12	.6	2	.1	4	.333333	8	.0274226
11	.55	1	.05	3	.25	7	.0137113
10	.5			2	.166666	6	.0089408
				1	.083333	5	.0074507
				Note. This Table of Dr: will also serve for Inches Months or Days		4	.0061171
				Penny n ^o	Decim:	3	.0050968
				10	.041666	2	.0042472
				9	.0375	1	.0035393
				8	.033333		
				7	.029166		
				6	.025		
				5	.020833		
				4	.016666		
				3	.0125		
				2	.008333		
				1	.004166		
				Grains	Decim:		
				20	.003472		
				10	.001736		
				9	.001562		
				8	.001389		
				7	.001215		
				6	.001042		
				5	.000868		
				4	.000694		
				3	.000521		
				2	.000347		
				1	.000173		
				$\frac{1}{2}$	.000086		

W. Woodcock sculp

Richards sculp

Decimal Tables

Measure and Time

Table 4. th		Table 5. th			Table 6. th	
Time 1 year y ^e Integer		Measure Liquid. Dry. Integer			Time 1 Day y ^e Integer	
Days	Decim:	1 Gall:	1 Quarter		Hours	Decim:
300	.821918	Ants	Decim:	Bricks	20	.833333
200	.547943	7	.875	7	10	.416666
100	.273973	6	.75	6	9	.375
90	.246373	5	.625	5	8	.333333
80	.219178	4	.5	4	7	.291666
70	.191781	3	.375	3	6	.25
60	.164383	2	.25	2	5	.208333
50	.136986	1	.125	1	4	.166666
40	.109589	q. ^{rs}	Decim:	Archs	3	.125
30	.082192	3	.09375	3	2	.083333
20	.054794	2	.0625	2	1	.041666
10	.027397	1	.03125	1		
9	.024637	Decim:	q. ^{rs} Pecks		Minutes	Decim:
8	.021918	.023437	3		50	.034722
7	.019178	.015625	2		40	.027777
6	.016438	.007812	1		30	.020833
5	.013699	Decim:	Pints		20	.013888
4	.010959	.005859	3		10	.006944
3	.008219	.003906	2		9	.00625
2	.005479	.001953	1		8	.005555
1	.002739	Coth Measure 1 yard y ^e Integer			7	.004861
		Quarters	Decim:		6	.004166
		3	.75		5	.003472
		2	.5		4	.002777
		1	.25		3	.002083
		Nails	Decim:		2	.001388
		3	.1875		1	.000694
		2	.125			
		1	.0625			
Wetters scrip!					Bickham foul!	



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CHAP. III.

Of the RULE of THREE.

THIS *Rule of PROPORTION* having been already sufficiently explain'd, both as to its nature and operation in *Vulgar Arithmetick*, 'tis needless to repeat here what has been there said. Only therefore observe, that instead of preparing our numbers, by reducing them into their lowest denomination, you must now bring their fractional parts into decimals. See the examples.

EXAMPLES.

EX. 1. If the price of 26 yards and $\frac{1}{2}$ of druggot
 3 l. 16 s. 3 d. what will 32 yards and $\frac{1}{4}$ come
 to?

The fractional parts of the numbers being reduced to decimals, and the question stated as usual, the work will stand as follows :

Yards. *l.* *Yards.*
If 26,5—3,8125—32,25

32,25
—
190625
76250
76250
114375

26,5)122,953125(4,63974
1695

The quotient valu'd
by the short rule for
money, makes 4 l. 12 s.
9 d $\frac{1}{2}$.

1053
—
2581
—
1962
—
1075
—
15

Ex. 2. What will the pay of 540 men amount to
at 1 l. 5 s. 6 d. per man?

Man. *l.* *Men.*
If 1—1,275—540
540
—
51000
6375
—

Answer, 688,500 or 688 l. 10 s.

For more examples, take those given in *Vulgar
Arithmetick*.

The Indirect RULE of THREE.

THE Indirect RULE of THREE is also the same here as in *Vulgar Arithmetick*, differing only in the preparation of the numbers.

E X A M P L E.

What must the penny-loaf weigh, when wheat 10s. the bushel, if, when at 6s. 8d. per bushel the penny-loaf weighs 5 oz. $\frac{1}{2}$?

If ,33333 — 5,5 — ,5
5,5

166665 *Ans.* oz. dwts. grs.
166665 3 13 7

,5)1,833315(
366663

Take also one Example in the DOUBLE RULE

If the carriage of $\frac{1}{2}$ a C. wt. 40 miles, comes to 6d. what will be the charge of carrying 16 C. 1000 miles?

First, If $,5\text{---},025\text{---}16,25$
 $,025$

025

8125

3250

,5),40625(

Miles.

Then, if $40 \frac{100}{100} = 8125 \frac{100}{100}$

100

40) 81,2500(

Facit 2,03125

07.

2 l. 0 s. 7 d $\frac{1}{2}$.



CHA

CHAP IV.

Of PRACTICE.

SOME questions in this rule may also be advantageously work'd by decimals ; as,

1st. When the given price is just two shillings, only separating the right hand figure for a decimal, the rest are pounds ; so in the

EXAMPLE.

5295 lb, at 2 s. per lb, comes to $\left\{ \begin{array}{l} 529,5 \\ \text{or} \\ 529 \text{ l. } 10 \text{ s.} \end{array} \right.$

2^d. When the price is any aliquot part of 2 s. 'tis only separating the right hand figure, as before, then dividing the number by such part.

EXAMPLE.

$\begin{array}{r} \text{oz.} \quad \text{d.} \\ 258,3 \text{ at } 8 \text{ per oz.} \\ \hline 8 \text{ d. } \frac{1}{3} 86,1 \text{ facit, or } 86 \text{ s.} \end{array}$

3^d. When the given price can be divided into such parts of two shillings, separate as before, take such parts.

I 3

EXAMPLE.

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E X A M P L E.

d. lb. d.
563,8 at 9 per lb.

6 $\frac{1}{4}$ 140,95
3 $\frac{1}{2}$ 70,475

facit 211,425 or 211 l. 8 s. 6 d.

4th. When the price is any number of shillings whatsoever, 'tis only multiplying by their half (which is the decimal of them) and the product is the answer.

E X A M P L E S.

lb. s. oz. s.
7296 at 12 s. per lb. 8264 at 15 s. per oz.
,6 ,75

facit 4377,6 or 4377 l. 12 s. 41320
57848

1.6198,00 facit.

5th. The requir'd price of any quantity may be found, by multiplying the said quantity by the decimal of the price given.

E X A M P L E.

lb. l. d.
8397 at 5 l. 13 s. 6 per lb.
5,675

41985
58779
50382
41985

facit 47652,975 or 47652 l. 19 s. 6 d.

6th. If there are fractional parts in the given quantity, they must be reduced to decimals, as well as the fractional parts of the price, as in the following

EXAMPLES.

3 $\frac{1}{2}$ at 12 per lb.
 $\begin{array}{r} 368,5 \\ ,65 \\ \hline 18425 \\ 22110 \\ \hline \end{array}$
facit 239,525
 or,
 239 l. 10 s. 6 d.

C. grs. lb. l. s. d.
 7368 3 14 at 1 12 6
 $\begin{array}{r} 7368,875 \\ 1,625 \\ \hline 36844375 \\ 14737750 \\ 44213250 \\ 7368875 \\ \hline \end{array}$
facit 11974,421875
 or,
 11974 l. 8 s. 5 d $\frac{1}{4}$.

The questions propos'd in *Vulgar Arithmetick*, all here also serve for more examples.





CHAP. V.

Of INTEREST and REBATE.

THESE rules having been already sufficiently explain'd in *Vulgar Arithmetick*, it only now remains to shew how much more advantageously they may be work'd by decimals. And *first*,

SIMPLE INTEREST.

1st. THE annual interest of any sum of money found by only multiplying the given principal by the interest of one pound for a year, which

$$\text{at } \left\{ \begin{array}{c} 4 \\ 5 \\ 6 \end{array} \right\} \text{ per Cent. is } \left\{ \begin{array}{c} ,04 \\ ,05 \\ ,06 \end{array} \right\}$$

found by dividing the rate of interest by 100.

EXAMPLE

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EXAMPLES.

1st. What's a year's interest of 536 ^{l.} at 5 per Cent? ^{l.}

$$\begin{array}{r} 536 \\ \times 5 \\ \hline \end{array}$$

facit 26,80 or 26 l. 16 s.

2^d. What will the interest of 2367 l. come to in a year, at 6 l. per Cent. per Ann.?

$$\begin{array}{r} 2367 \\ \times 6 \\ \hline \end{array}$$

facit 142,02 or 142 l. 0 s. 5 d.

3^d. Thus a year's interest of any sum being found, 'tis only multiplying that sum by 2, 3, 4, &c. and the product is the interest of the said sum for 2, 3, 4, or more years:

EXAMPLE.

What's the interest of 623 for 3 years, at 4 per Cent.?

$$\begin{array}{r} 623 \\ \times 4 \\ \hline 2492 \text{ Interest for 1 year.} \\ \times 3 \\ \hline 7476 \\ \text{or, } \left. \begin{array}{l} 7476 \\ 2492 \end{array} \right\} \text{Interest for 3 years.} \\ 74 \text{ l. } 15 \text{ s. } 2 \text{ d } \frac{1}{2}. \end{array}$$

4^d. Again, If the time requir'd be months, 'tis only dividing the year's amount by such parts as the given months are of a year.

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EXAMPLES.

EXAMPLES.

Ex. 1st. What is the interest of 539 l. for four months, at 5 l. per Cent. per Annum?

$$\begin{array}{r}
 \text{l.} \\
 539 \\
 ,05 \\
 \hline
 26,95 \text{ Interest for a year.} \\
 \text{Months} \\
 4 \frac{1}{2} \quad 8,9813 \left. \begin{array}{l} \text{or,} \\ 8 \text{ l. } 19 \text{ s. } 8 \text{ d.} \end{array} \right\} \text{facit.}
 \end{array}$$

Ex. 2^d. What will the interest of 729 l. 10 s. 6 d. come to in a year and 5 months, at 6 per Cent. per Annum?

Note, If there are any odd shillings, pence, and farthings in the principal sum, they must be reduced to decimals, as in this example.

$$\begin{array}{r}
 729,515 \\
 ,06 \\
 \hline
 \text{Months} \quad 43,77150 \text{ Interest for a year.} \\
 4 \frac{1}{2} \quad 14,59050 \text{ Interest for 4 months.} \\
 1 \frac{1}{2} \quad 3,647625 \text{ Interest for 1 month.} \\
 \hline
 \text{facit} \quad 62,009625 \text{ Total Interest.} \\
 \text{or,} \\
 62 \text{ l. } 0 \text{ s. } 2 \text{ d. } \frac{1}{4}.
 \end{array}$$

4th. But if it be required to find the interest of a sum of money for any number of days, you must either multiply the year's amount by the number of days propon'd, and divide the product by 365, or (which will be sometimes shorter) divide the year's

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by the

What
at 6 l. p

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ated as a

year's amount by 365, and multiply the quotient by the number of days propos'd.

EXAMPLE.

What interest will 563 l. amount to in 126 days.
at 6 l. per Cent. per Annum.

$\begin{array}{r} \text{1.} \\ 563 \\ ,06 \\ \hline 33,78 \\ 126 \\ \hline 20168 \\ 6756 \\ \hline 3178 \end{array}$	$\begin{array}{r} 365)33,78000(,09254 \\ \hline 11930 \\ \hline 2000 \\ \hline 1750 \\ \hline 290 \\ ,09254 \\ 126 \\ \hline 55524 \\ 18508 \\ \hline 9254 \\ \hline 11,66004 \end{array}$
$\begin{array}{r} 365)4256,28(11,661 \\ \hline 606 \\ \hline 2412 \\ \hline 2228 \\ \hline 180 \\ \hline 15 \end{array}$	

But for the more ready casting up the Interest of money for days, it will be necessary to have the interest of 1 l. for 1 day, at all rates ready calculated as a table or standard. Thus:

The INTEREST of 1 l. for 1 Day.

At {	1	per Cent. is {	,00002739726
	2		,00005479452
	3		,00008219178
	4		,00010958904
	5		,00013698630
	6		,00016438356
	7		,00019178082
	8		,00021917808
	9		,00024657534
	10		,00027397260

The interest of 1 l. for one day, is thus found; the given rate or interest of 100 l. for a year, being divided by 100, quotes the interest of 1 l. for a year; which again divided by 365, gives the required interest of 1 l. for 1 day; which, when multiply'd into both the number of days, and the principal sum, produces the interest for the time required.

EXAMPLE.

What's the interest of 560 l. for 60 days, at 5 l. per Cent. per Annum?

,0001369863
560 Principal.

82191780
6849315

767123280
60 Days.

Payt 4,6027396800 or 4 l. 12 s. 0 d.

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A Table

Shewing the Number of Days from any day in any Month to the same day in any other Month.

From	Jan:	Feb:	Mar:	April:	May:	June:
To	Feb: 31	Mar: 28	April: 31	May: 30	June: 31	July: 30
	Mar: 59	April: 59	May: 61	June: 61	July: 61	Aug: 61
	April: 90	May: 89	June: 92	July: 91	Aug: 92	Sep: 92
	May: 120	June: 120	July: 122	Aug: 122	Sep: 123	Oct: 122
	June: 151	July: 150	Aug: 153	Sep: 153	Oct: 153	Nov: 153
	July: 181	Aug: 181	Sep: 184	Oct: 183	Nov: 184	Dec: 183
	Aug: 212	Sep: 212	Oct: 214	Nov: 214	Dec: 214	Jan: 214
	Sep: 243	Oct: 242	Nov: 245	Dec: 244	Jan: 245	Feb: 245
	Oct: 273	Nov: 273	Dec: 275	Jan: 275	Feb: 276	Mar: 275
	Nov: 304	Dec: 303	Jan: 306	Feb: 306	Mar: 304	April: 304
	Dec: 334	Jan: 334	Feb: 337	Mar: 334	April: 335	May: 334
	Jan: 365	Feb: 365	Mar: 365	April: 365	May: 365	June: 365

From	July:	Aug:	Sep:	Oct:	Nov:	Dec:
To	Aug: 31	Sep: 31	Oct: 30	Nov: 31	Dec: 30	Jan: 31
	Sep: 62	Oct: 61	Nov: 61	Dec: 61	Jan: 61	Feb: 61
	Oct: 92	Nov: 92	Dec: 91	Jan: 92	Feb: 92	Mar: 90
	Nov: 123	Dec: 122	Jan: 122	Feb: 123	Mar: 120	April: 120
	Dec: 153	Jan: 153	Feb: 153	Mar: 151	April: 151	May: 151
	Jan: 184	Feb: 184	Mar: 184	April: 182	May: 184	June: 184
	Feb: 215	Mar: 212	April: 212	May: 212	June: 212	July: 212
	Mar: 243	April: 243	May: 242	June: 243	July: 242	Aug: 243
	April: 274	May: 273	June: 273	July: 273	Aug: 273	Sep: 274
	May: 304	June: 304	July: 303	Aug: 304	Sep: 304	Oct: 304
	June: 335	July: 334	Aug: 334	Sep: 333	Oct: 334	Nov: 335
	July: 365	Aug: 365	Sep: 365	Oct: 365	Nov: 365	Dec: 365

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Table.

The year being divided into 12 unequal months, the following verses may serve as a memorandum of the number of days in each, viz.

*Thirty days hath September,
April, June, and November;
February hath twenty eight alone,
And all the rest have thirty one.*

But without any farther trouble, the opposite table shews, by inspection only, the number of days from any day in any month, to the same day in any other month. As suppose the number of days, betwixt the 4th of *February* and the 4th of *September*, were requir'd; looking in the column under *February* for *September*, against that month will stand, 212, the number of days betwixt the said times; so again, from the 8th of *June* to the 8th of *October*, appears by the table to be 122 days, that being the number against *October*, in the column under *June*.

Note, If the given days are different, 'tis only adding, or subtracting their inequality to, or from the tabular number. Thus had the first example been from the 4th of *February*, to the 8th of *September*, it had been 4 days more than 212, viz. 216; or had the time, in the last example, been from the 8th of *June*, to the 4th of *October*, it had been 4 days less than 122, viz. 118.

Note also, If the time exceeds a year, 365 days must be added. Thus from the 4th of *February* 1714, to the 4th of *September* 1715, will be found to be 377 days, the sum of 212, and 165. See the Table.

COMPOUND

COMPOUND INTEREST.

1st. **T**O find the amount of any sum, at any rate of compound interest, for any number of years.

R U L E.

Multiply the rate, that is, the amount of 1 *l.* for a year, which at 6 *per Cent.* is 1,06, at 5 *per Cent.* 1,05, &c.) so often into it self, as are the number of years propos'd, wanting one; and the last product multiply'd by the principal, will give the amount required.

E X A M P L E.

What's the amount of 500 *l.* forborn four years, at 4 *l. per Cent. per Annum*?

$$\begin{array}{r}
 1,04 \\
 1,04 \\
 \hline
 1,0816 \\
 1,04 \\
 \hline
 1,124864 \\
 1,04 \\
 \hline
 1,16985856 \\
 500 \\
 \hline
 \hline
 \end{array}$$

fact 594,91929600

And thus is the first interest-table (which shews the amount of 1 *l.* for any number of years, under 11, at the rates of 5 and 6 *l. per Cent. per Annum*) form'd.

The use of which is plain and easy; for multiply- ing the figures standing against the number of years requir'd; and under the given rate, by the propos'd principal, the product is the amount desired.

Thus,

Thus, if the amount of 40 *l.* in 20 years, at 5 *l.* per Cent. per Annum, were requir'd, 2,65329 the tabular number, multiply'd by 40 *l.* the principal sum, gives 106,13160, or 106 *l.* 2 *s.* 7 *d.* $\frac{1}{2}$, the Answer. And so any other.

2d. To find the amount of any annuity, or yearly pension, forborn any number of years whatsoever, at any rate of compound interest.

R U L E.

Multiply the first yearly payment by the rate, and to the product add the second yearly payment, the sum is the amount in 2 years, which, multiply'd again by the rate, the product, with the addition of the third yearly payment, is the amount for 3 years, &c.

E X A M P L E.

What will a pension of 30 *l.* per Annum amount to, being forborn 4 years, at 5 *l.* per Cent. per Annum?

	1.	
First yearly payment	30	
multiply'd by the rate	1,05	
	31,50	
Second yearly payment added	30	
	gives 61,50	the amount in 2 years.
	64,5750	
Third yearly payment	30	
	94,5750	Amount in 3 years.
	99,103750	
Fourth yearly payment	30	
	129,303750	Amount in 4 years.

And

And after this manner is the second interest table (which shews the amount of 1 l. annuity for any number of years under 33, at the rates of 5 and 6 l. per Cent. per Annum, compound interest) compos'd.

But this second table may more easily be form'd from the first, thus: the first line of this second table, or first year's amount being known, add to it the first line of the first table, the sum is the amount for 2 years, or the second line of this table; to which again adding the second line of the first table, you have the third of this, or the amount for three years, &c.

EXAMPLE at 6 l. per Cent.

1.		
1,	=====	First year of the second table.
1,06	=====	First of the first table.
2,06	=====	Second year of the second table.
1,1236	=====	Second year of the first.
3,1836	=====	Third year of the second.
1,19101	=====	Third year of the first.
4,37461	=====	Fourth year of the second, &c.

The use of this second table is in the same manner as the first, as suppose the foregoing question for an example.

EXAMPLE.

4,31012 } the amount of 1 l. annuity for 4
 years, at 5 l. per Cent.
 multiply'd by 30 the yearly sum,

 gives 129,30360 as before.

REBATE or DISCOMPT.

REBATE, according to simple interest, having been sufficiently explain'd in *Vulgar Arithmetick*, I shall only here set down one example work'd decimally the common way. And, to compleat the section, add Mr. *Hutton's* new method of finding the present worth and discmpt of money, as deliver'd in his *System of Arithmetick*, pag. 188.

EXAMPLE.

What discmpt must be allow'd on a bill of 500 l. paid 20 days before it is due, rebate at 5 l. per Cent. per Annum? And what present money must be paid?

The common way.

Days.	l.	Days.
If 365	— 5 —	20
	5	
	—	
	365) 10000 (27397	
	—	
	2700	
	—	
	1450	
	—	
	3550	
	—	
	1650	
	—	
	95	

Then

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Then for the discompt :

$$\text{If } 100,27397 \text{ --- } ,27397 \text{ --- } 500$$

$$100,27397) 136,98500 (1,3661 \text{ discompt.}$$

$$\underline{36711030}$$

$$\underline{66288390}$$

$$\underline{61240080}$$

$$\underline{10756980}$$

$$629593$$

or for the present worth,

$$\text{If } 100,27397 \text{ --- } ,100 \text{ --- } 500$$

$$100,27397) 50000,000 (498,6338 \text{ present worth.}$$

$$\underline{98904120}$$

$$\underline{86575470}$$

$$\underline{63562940}$$

$$\underline{33985580}$$

$$\underline{19033890}$$

$$\underline{89516990}$$

$$9297814$$

Mr. HATTON's New Method.

1. For the present worth, multiply the days in a year, the principal given, and 100 into each other, for a dividend; and add the product of 365 by 100, to that of the days multiply'd into the rate given for a divisor, so the quotient arising is the answer.

Thus

Thus in the foregoing example $365 \times 500 \times 100 = 18250000$ for the dividend, and $365 \times 100 \div 20 \times 5 = 36600$ for the divisor.

See the work.

$366 \overline{) 182500} \overline{) 182500} \overline{) 182500} \overline{) 182500} \overline{) 182500}$ present worth.

$$\begin{array}{r}
 3610 \\
 \hline
 3160 \\
 \hline
 2320 \\
 \hline
 1240 \\
 \hline
 1420 \\
 \hline
 322
 \end{array}$$

2. For the *discompt*, multiply the rate principal, and days given together for a dividend; and proceeding as before directed for a divisor, the quotient will be the answer.

Thus keeping still the same example, $5 \times 500 \times 20 = 50000$ for the dividend, and $365 \times 100 \div 20 \times 5 = 36600$ for a divisor, as before.

See the work.

$366 \overline{) 50000} \overline{) 50000} \overline{) 50000} \overline{) 50000} \overline{) 50000}$ discompt.

$$\begin{array}{r}
 1340 \\
 \hline
 1410 \\
 \hline
 2140 \\
 \hline
 440 \\
 \hline
 74
 \end{array}$$

The

The truth of both these methods appears, not only by their agreement, but also by the addition of the present worth, and discompt, found, which, as near as can be, makes the principal.

Thus 498,6338 the present worth,
with 1,3661 the discompt,

makes 499,9999 = 500 l. the given principal.

REBATE at COMPOUND INTEREST.

1st. **T**O find the present worth of any sum to be paid at any number of years to come, rebate being made at any rate of compound interest.

R U L E.

Divide the principal continually so many times by the rate, as are the number of years propos'd, and the last quotient is the answer.

E X A M P L E.

What is the present worth of 1 l. to be paid at the end of 6 years, rebate at 6 l. per Cent. per Ann. compound interest ?

1.				years.	
The principal 1,		divided by		present worth at the end of	
	.943396		.943396		1
	.889997		.889997		2
	.839619		.839619		3
	.792094		.792094		4
	.747258		.747258		5
		gives	.704960		6

And thus is the third table made, which shews the present worth of 1 l. due at any number of years to come, under 33, rebate at 5 and 6 l. per Cent. per Annum, compound interest ; the use of which is, as

as the other tables, only by multiplying the present worth of 1 *l.* by the given principal, and the product is the present worth required.

Thus suppose in the example above, the principal had been 200 *l.* the present worth of 1 *l.* viz. .704960 multiply'd by 200, gives 140,992000, the present worth of 200 *l.* and so any other.

2d. To find the present worth of any annuity, or yearly pension, to continue any number of years; rebate being made at any rate, *per Cent. per Annum*, compound interest.

R U L E.

Find by the foregoing rule the present worth of the propos'd annuity, for 1, 2, 3, or so many years as are demanded; and the sum of those respective present worths will be the value of the annuity; that is, the first and second of those values, added together, will be the present worth for two years; and the first, second, and third, added, for three years, &c.

E X A M P L E.

What's the present worth of an annuity of 50 *l.* to continue four years, rebate at 5 *l.* *per Cent. per Annum*?

$$\begin{array}{rcl}
 1. & & \\
 50, & & \\
 47,61904 & \left. \begin{array}{l} \text{divided by } 1,05 \\ \text{the rate, gives} \end{array} \right\} & \begin{array}{l} 47,61904 \\ 45,35147 \\ 43,19187 \\ 41,13512 \end{array} \\
 45,35147 & & \\
 43,19187 & & \\
 \hline
 \end{array}$$

The total of which is 177,29750 the present (worth requir'd.

And after this manner is made table the fourth, which shews the present worth of 1 *l.* annuity, to

to continue any number of years under 33, rebate at 5 and 6 l. per Cent. per Annum, compound interest. Which table, as the others, will give the present worth of any sum whatsoever, by only multiplying the value of 1 l. by the sum propos'd. Thus suppose the foregoing example.

E X A M P L E.

3,54595 the present worth of 1 l. An. for 4 yrs.
multipl'y'd by 50 the Annuity propos'd.

gives 177,29750 as before.

The fourth table may be also form'd from the third, thus;

The first year is the same in both; the first and second years of the third, added together, make the second of the fourth; the second year of the fourth, added to the third year of the third, makes the third of the fourth table, &c. See the example.

E X A M P L E at 6 l. per Cent.

.94339 * The first year of both tables.
.88999 The second year of the third table.

1,83339 The second year of the fourth.
.83961 The third year of the third.

2,67300 The third year of the fourth,
&c.

* Note, 1 is added to the right hand figure, in consideration of the places omitted.

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More

Decimal Tables

showing

TABLE 1. The Amount of one Pound for any Number of Years under 33. at 4 per Cent. Rate of 3.86 per Cent. Com. Interest.

TABLE 2. The Amount of 1st Annunity for any Number of Years under 33. at 4 per Cent. Rate of 3.86 per Cent. Com. Interest.

£ Annu 6		Years	£ Annu 6	
1.05000	1.06000	1	1.00000	1.00000
1.10250	1.12360	2	2.03000	2.06000
1.15762	1.19101	3	3.15250	3.18360
1.21550	1.26247	4	4.31012	4.37461
1.27628	1.33822	5	5.52363	5.63709
1.34009	1.41852	6	6.80191	6.97332
1.40710	1.50363	7	8.14200	8.39383
1.47743	1.59384	8	9.54910	9.89746
1.55132	1.68948	9	11.02636	11.49131
1.62889	1.79084	10	12.57789	13.18059
1.71034	1.89829	11	14.20678	14.97164
1.79583	2.01219	12	15.91712	16.86994
1.88563	2.13292	13	17.71298	18.88213
1.97993	2.26090	14	19.59863	21.01500
2.07889	2.39655	15	21.57856	23.27359
2.18257	2.54033	16	23.65719	25.67252
2.29101	2.69277	17	25.83436	28.21208
2.40402	2.85434	18	28.11338	30.90463
2.52169	3.02559	19	30.50000	33.75999
2.64329	3.20713	20	33.00393	36.78559
2.76896	3.39956	21	35.71923	39.99252
2.89820	3.60353	22	38.55021	43.39220
3.03132	3.81955	23	41.50047	46.99582
3.16850	4.04893	24	44.57099	50.81537
3.31033	4.29187	25	47.77279	54.86411
3.45667	4.54938	26	51.11343	59.16618
3.60740	4.82234	27	54.60912	63.73070
3.76263	5.11168	28	58.26258	68.57281
4.11613	5.41838	29	62.07271	73.69979
4.32194	5.74349	30	66.04384	79.12810
4.53804	6.08810	31	70.17607	84.86167
4.76494	6.45338	32	75.29883	90.89977

Decimal Tables

showing

TABLE 3. The present Worth of £1 due at any number of years to come under 33. Rates at 5 & 6 per cent. Ann. Interest

TABLE 4. The present Worth of £1 Annuity to continue any number of Years under 33. Rates at 5 & 6 per cent. Ann. Interest

Rates		Years	Rates	
5	6		5	6
0.923801	0.943396	1	0.923801	0.943396
0.907030	0.899996	2	1.83941	1.833333
0.891038	0.859619	3	2.72324	2.67301
0.872702	0.822093	4	3.57493	3.46310
0.853326	0.787238	5	4.3947	4.21236
0.83213	0.754966	6	5.17369	4.91732
0.810682	0.663037	7	5.91837	5.58238
0.7676830	0.627412	8	6.6321	6.20979
		9	7.31782	6.80169
0.644600	0.591898	10	7.97213	7.36608
0.613013	0.558394	11	8.60641	7.90607
0.584670	0.526787	12	9.216323	8.42304
0.558637	0.496969	13	9.80337	8.91268
0.530321	0.468830	14	10.36864	9.38498
0.503068	0.442381	15	10.91303	9.83123
0.481017	0.417263	16	11.43777	10.2529
0.458111	0.393677	17	11.94206	10.6516
0.436306	0.371404	18	12.42688	11.02760
0.413320	0.350343	19	12.89333	11.3811
0.393734	0.330313	20	13.34221	11.71292
0.376889	0.311804	21	13.773113	12.02407
0.362942	0.294133	22	14.18600	12.31458
0.347849	0.277303	23	14.58087	12.58338
0.333571	0.261797	24	14.95864	12.83033
0.310067	0.246978	25	15.319394	13.05333
0.293302	0.232998	26	15.66318	13.25316
0.277240	0.219070	27	15.9903	13.43033
0.267848	0.205368	28	16.3012	13.58616
0.255093	0.193030	29	16.5957	13.72072
0.242946	0.184336	30	16.87407	13.83483
0.231377	0.174110	31	17.139281	13.92908
0.220339	0.164254	32	17.39267	14.00404
0.209863	0.154956			

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More *EXAMPLES* for the Exercise of the foregoing
R U L E S and T A B L E S.

Ex. 1st. What will 1000 *l.* increase to, forborn 22 years, at 5 *per Cent. per Annum* compound interest?

Answer, 2925 *l.* 5 *s.* 2 *d.* $\frac{1}{2}$.

Ex. 2^d. What would an annuity of 520 *l.* forborn 17 years, at 6 *per Cent.* compound interest, amount to?

Answer, 14670 *l.* 13 *s.* 11 *d.* $\frac{1}{4}$.

Ex. 3^d. What present money would discharge a debt of 7500 *l.* to be paid at the end of 10 years, rebate being made at 5 *l. per Cent. per Annum*?

Answer, 4604 *l.* 6 *s.* 11 *d.* $\frac{1}{4}$.

Ex. 4th. What is a yearly rent of 65 *l.* to continue 30 years, worth in ready money, rebate being made at 6 *per Cent. per Annum* compound interest?

Answer, 894 *l.* 14 *s.* 3 *d.* $\frac{1}{4}$.

Ex. 5th. What is the present worth of a reversion of a lease of 500 *l. per Annum*, to continue 20 years, but not to commence 'till after the end of five years, allowing the purchaser 6 *per Cent.* compound interest?

Note, To work questions of this nature, there must be two distinct operations, *viz.*

1st. The present worth of the propos'd annuity, for the given time of its continuance, must be found as if it were immediately to commence. Then,

2^d. See what principal, forborn at the given interest, would, in the time of the reversion, amount to the aforesaid present worth, and that principal will be the present worth of such annuity in reversion.

Thus

Thus by table the 4th, the present worth of 1 l. per Annum to continue 20 years, at 6 l. per Cent. is found to be

which multiply'd by the propos'd annuity, $\frac{11,46992}{500}$
gives 5734.9600

Then in the first table against 5 years, the time of the reversion, under the same rate of interest is found 1.33822 the amount of 1 l. in the said time; therefore

If 1.33822 arise from 1 l. from what principal does 5734.960000 arise?

Answer, 4285.5135, or 4285 l. 10 s. 3 d $\frac{1}{4}$.

Or It may more easily be done by first finding the present worth of the annuity for the whole time both of possession and reversion, and then deducting the present worth of the possession; the remainder will be the present worth of the reversion. Thus 12.78335, the tabular number, against 25 years, the whole time being multiply'd by 500, gives 6391.67500, from which subtracting 2106.18000 the produce of 4.21236 the tabular number against 5 years, the time of the possession, the remainder 4285.49500 is the value of the reversion: Within a trifle the same answer as before.

FREEHOLD ESTATES.

TO find the present worth of any annual rent, to continue for ever, commonly call'd fee-simple.

RULE.

R U L E.

Divide the propos'd rent by the interest of 1 l. for one year, (which at 5 l. per Cent. is .05, or at 6 per Cent. .06, as before shewn) and the quotient is the present value of the estate.

E X A M P L E.

What is an estate of 200 l. per Annum, to continue for ever, worth in ready money, allowing the purchaser 5 l. per Cent. per Annum compound interest ?

,05)200,000

Answer, 4000 l.

Or it may be done by multiplying the fee-simple of 1 l. by the yearly rent propos'd.

The fee-simple of 1 l. per Annum compound interest, at

1 per Cent. 2 per Cent. 3 per Cent. 4 per Cent. 5 per Cent.
100,00000 50,00000 33,33333 25,00000 20,00000

6 per Cent. 7 per Cent. 8 per Cent. 9 per Cent. 10 per Cent.
16,66667* 14,28571 12,50000 11,11111 10,00000

* Note, In contracting a line of decimals from many to fewer places, 1 is added to the last figure retain'd, if the next omitted figure exceedeth 5; which rule hath been all along observ'd in the tables, &c.

The foregoing question work'd by the last rule.

The fee-simple of 1 l. per Ann. at 5 l. per Cent. is 20
Which multiply'd by the yearly rent, 200

gives as before, 4000

K

I will

I will now only add another example or two, for the reader's exercise, and conclude this rule.

1. *A* has the possession of an estate of 120 *l. per Annum*, to continue 20 years; *B* has the reversion of the same, from that time for ever. What must *A* give *B*, if he would purchase his reversion? And what must *B* give *A*, if he would buy his possession, accounting 6 *per Cent.* compound interest in each case?

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Answer, { <i>A's</i> possession is worth	1491	1	9½
{ <i>B's</i> reversion is worth	675	11	6½

2. *A* agrees with *B* for an annuity of 600 *l. per Annum*, to continue 25 years, to give him the present worth of it, at 6 *l. per Cent. per Annum*; but not having money enough by him, offers to make over to him a freehold estate of 12 *l. per Annum* at the same interest, What money besides will pay his purchase?

Answer, 7470 *l. os. 2d ½.*





CHAP. VI.

Of FELLOWSHIP.

Without taking any notice of the common way of working this rule, observe this much more compendious method.

R U L E.

Divide the whole gain, or loss, by the whole stock, and multiply the quotient by each man's particular stock, the several products are the respective gains of each.

Note, It is necessary to place so many cyphers on the right hand of your dividend, as will bring out six or seven places of decimals in the quotient.

E X A M P L E.

Suppose *A*, *B*, and *C* trading together, *A*. puts in 1000 *l*. *B* 700 *l*. and *C*. 1200 *l*. the whole gain is 836 *l*. What share of it belongs to each?

l.
A 500
B 700
C 1200

Total Stock 2400) 836,000000 (,348333

11600

20000

• 8000

• 8000

• 8000

• 800

,348333

500

,348333

700

,348333

1200

A's share 174,166500 B's 243,833100 C's 417,999600

B's share 243,833100

C's share 417,999600

Tot. gain 835,999200, or 836 l. as near as can be.

See another EXAMPLE with Time.

A puts into company 525 l. 10 s. for six months
B 382 l. 15 s. for eight months, and C. 1000 l. for
four months; they gain in all 1286 l. 12 s. How
much is that for each?

	<u>525,5</u> 6	<u>382,75</u> 8	<u>2000</u> 4
A's stock	3152,0	3062,00	4000
B's stock	3062		
C's stock	4000		
10215)1286,600000(,125952			
	<u>,125952</u> 3152	<u>,125952</u> 3062	<u>,125952</u> 4000
A's gain	397,126656	385,665024	503,808000
B's	385,665024		
C's	503,808000		

Total 1286,599680 or 1286 l. 12 s. as near as can be.





C H A P. VII.

Of EVOLUTION, or EXTRACTION
of ROOTS.

The S Q U A R E R O O T.

EXTRACTION of the S Q U A R E R O O T is finding the side of a square figure; or, numerically speaking, it is finding what number, multiply'd by it self, will produce the number given. Thus the square root of 16 is 4, 4 times 4 making 16.

What a square is, may be seen in the following figure, which being divided every way into 3 equal parts, its whole content or square is 9, and its side or root is 3.



Square

Square Numbers are either single or compound.

A single square number is always less than 100, being produc'd by the multiplication of some one single figure by it self, as 16 from 4, &c. So that the root of any single square maybe found in the annex'd table, always taking the root of the next less square, for any number not there inserted, as for 26 take 5, or 3 for 10, &c.

Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound square number being compos'd by the multiplication of two or more figures by themselves, always exceeds 100; as 144, which is 12 times 12; or 195, which is 15 times 15, &c.

If the root therefore is express'd by two figures, its square must at last consist of three, for the least root express'd by two figures is 10, whose square is 100. And if the root has three figures, its square must at least have five. If four, the square has seven, &c. so that you cannot augment the root one figure, but you encrease the square two.

To find the root of any compound square number, as suppose 2704,

1st. You must distinguish it into single squares, by placing dots over every other figure, beginning at the right hand, thus,

2704

And so many dots as happen, so many places will the root consist of, which in this example are two.

K 4

adly.

2^{dly}. Drawing a crooked line on the right hand of your number, as in division, find the root of your first single square, and place it in the quotient.

See the work :

$$\begin{array}{r} \dot{2}70\dot{4}(5 \end{array}$$

3^{dly}. Placing the square of the root found, under the first single square, subtract, and set down the remainder; bringing down to it, the next single square, call the line a *Resolvend*.

$$\begin{array}{r} \dot{2}70\dot{4}(5 \\ 25 \\ \hline \end{array}$$

· 204 *Resolvend*.

4^{thly}. Drawing another crooked line on the left hand of the said resolvend, place beyond it the double of the quotient, in form of a divisor.

$$\begin{array}{r} \dot{2}70\dot{4}(5 \\ 25 \\ \hline 10) \cdot 204 \end{array}$$

5^{thly}. Dividing the resolvend, all but the unit's place, by the said divisor, (which, in this example, is the 10's in 20) set down the number of times it goes, (*viz.* 2) both in the quotient, and on the right hand of the divisor :

$$\begin{array}{r} \dot{2}70\dot{4}(52 \\ 25 \\ \hline 102)204 \text{ *Resolvend*.} \end{array}$$

6^{thly}.

6^{thly}. Multiplying the whole divisor by the figure last plac'd in the quotient, set down the product under, and subtract it from the resolvend.

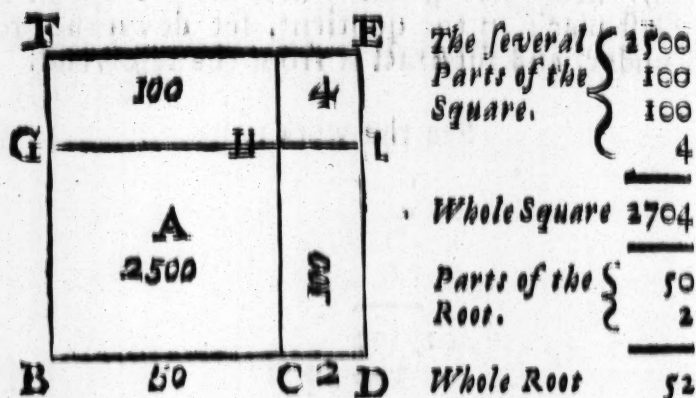
See the work :

$$\begin{array}{r}
 2704 \overline{)52} \\
 \underline{25} \\
 102 \overline{)104} \\
 \underline{104} \\
 0
 \end{array}$$

Note, If the divisor, multiply'd by the quotient-figure, gives a product greater than the resolvend, 'tis false, and must be rectify'd by a smaller quotient-figure.

Extraction of the square root being a matter rather lineal than numeral, the reason of the rules above given for its operation, will best appear from the following figure : Wherein let it be consider'd, that if the root *B. C.* of the square *A*, be augmented by the magnitude *C. D.* the said square will be enlarged by the figure *C. D. E. F. G. H. I.* equal to an oblong or long square, whose length shall be the root *B. C.* twice taken, with the line *C. D.* and breadth the same line *C. D.*

N. B. The particular contents of the several parts of the figure are as therein set down.



In the operation therefore, having taken the nearest root of the first square 27, viz. 5, and placed it in the quotient, from the place it stands in, (viz. the second) it must be accounted 50, the square of which, viz. 2500 (mark'd *A* in the figure) being subtracted from the given number 2704, there remains 204, equal to the figure *C. D. E. F. G. H. I.* In order to find the content of which, I double the root *B. C.* viz. 50, for the two bases *C. H.* and *H. G.* which making 100, I set it down as a divisor; and to find the height of the said figure, viz. *C. D.* I seek how often the said divisor is contain'd in the number 204, and finding it twice, I set down two in the quotient for the said height, and also in the unit's place of my divisor for the base *H. I.* Then multiplying the bases by the height, I have the content of the whole annex'd figure *C. D. E. F. G. H. I.* viz. 204, exactly equal to the number remaining, after the subtraction of the first great square 2500; so that the number given, viz. 2704, appears to be a perfect square, whose root or side is exactly 52.

Note. As many single squares as are in the number propos'd, so many times must the work of the three last rules be repeated; every square being to be brought down, as directed in the third rule.

EXAMPLE.

EXAMPLE.

What's the Root of 582169)763 *Answer.*

$$\begin{array}{r}
 49 \\
 \hline
 146 \overline{) .921} \text{ Resolvend.} \\
 \underline{876} \\
 1523 \overline{) .4569} \text{ Resolvend.} \\
 \underline{4569} \\
 \hline
 0
 \end{array}$$

In this example, to 45 the second remainder, 69 the next square, being brought down, makes 4569 for a new resolvend; and 76, the quotient-figures, being doubled for a new divisor, makes 152, which going three times in the resolvend, 3 is plac'd both in the quotient, and on the right hand of the divisor, which being then multiply'd by 3, gives 4569, equal to the resolvend; so that nothing remains.

P R O O F.

The proof of this extraction, is only multiplying the root found by it self, which, if right, will produce the square number given. *Note,* If any thing remains, it must be taken in.

To find the fractional Part of the Root,

Add to the number given, a competent number of pairs of cyphers, and proceed by the foregoing rules: As suppose 75896 were given, for an example.

EXAMPLE.

$$\begin{array}{r}
 75896,000000(375,492 \\
 4 \\
 \hline
 47)358 \\
 329 \\
 \hline
 145)2996 \\
 2725 \\
 \hline
 5504)27100 \\
 22016 \\
 \hline
 55089)508400 \\
 495801 \\
 \hline
 550982)1259900 \\
 1101964 \\
 \hline
 157936
 \end{array}$$

Note, As many pairs of cyphers as are added, for many places of decimal parts will there be in the root.

To extract the SQUARE ROOT of a VULGAR FRACTION.

Find the root of the numerator for a numerator and let the root of the denominator be its denominator; thus the square root of $\frac{16}{9}$ is $\frac{4}{3}$, and of $\frac{9}{4}$ is $\frac{3}{2}$, &c.

The root of a mix'd number is also found after the same manner, being first reduc'd into an improper

per fraction; thus $6\frac{1}{4}$, reduc'd, makes $3\frac{1}{2}$, the root of which is $\frac{1}{2}$, or $2\frac{1}{2}$.

But if either a proper fraction, or a mix'd number, be incommensurable to its root, to extract it, reduce the fraction to a decimal of an even number of places, and extract the root as if it were a whole number, thus $\frac{1}{4}$, reduc'd to a decimal, will be .25, the square root of which is .5, &c. or had it been $4\frac{1}{4}$, which reduc'd, is 4.25, the root would be 2.06, &c.

The CUBE ROOT.

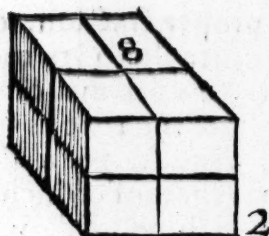
EXtraction of the CUBE ROOT, is finding the side of a solid figure, whose length, breadth, and depth, are equal; or, numerically speaking, it is finding what number, multiply'd twice into its self, will produce the number given; thus the Cube Root of 64 is 4, 4 times 4 being 16, and 4 times 16 make 64.

What a Cube is, may be something explain'd by the following figure, wherein 8 dice are so dispos'd, that there are 2 every way, that is, 2 in length, 2 in breadth, and 2 in height; so that 8 is the cube, and 2 the root.

See

[218]

See the Figure.



Cube-numbers are either single or compound.

A single cube-number is always less than 1000, being produc'd by the multiplication of one single figure, first by it self, and then by its product, as 125 from 5, &c. so that the root of any single cube may be found in the annex'd table; always remembring to take the root of the next less cube for any number not there inserted, as for 513 take 8, or 6 for 220, &c.

Cubes	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound cube-number being compos'd by the multiplication of 2 or more figures, first by themselves, and then by their product, always exceeds 1000, as 1728 from 12, or 15625 from 25, &c.

If therefore the root is express'd by two figures, its cube must at least consist of four; for the least root express'd by two figures is 10, whose cube is 1000; if the root has three figures, its cube must at least have seven, &c. So that you cannot augment the root one figure, but you encrease the cube three.

Therefore

Therefore to find the root of any compound cube-number, as suppose 15625.

1st. You must distinguish it into single cubes, by placing dots over every third figure, beginning from the right hand, thus:

15625

And so many dots as happen, so many places will the root consist of, which, in this example, are two.

2^{dly}. Drawing a crooked line on the right hand of your number, as in division, set down as a quotient the root of your first single cube, which, in this example, is 2.

3^{dly}. Placing 8, the cube of 2. the root found, under 15 the first single cube, subtract, and to the remainder 7 bring down 625 the next single cube, and it will make 7625, which call a Resolvend.

See the work :

15625(2
8

7625 Resolvend.

4^{thly}. Draw a line under the resolvend, and tripling the square of the root 2, set the said triple square, (*viz.* 12) under the resolvend; so that units in the said triple square, may stand under the place of hundreds in the resolvend.

5^{thly}.

5^{thly}. Subscribe also the triple of the root 2 (*viz.* 6) so that units, in this, may stand under the place of tens in the resolvend.

6^{thly}. The triple square of the root, and triple root, being plac'd as directed, draw a line under them, and add them together in the order they are plac'd; the sum 126 is a divisor.

7^{thly}. Accompting all the resolvend (except the place of units, (*viz.* 762) a dividend, seek how often the divisor (126) is contain'd in it, and place the number of times (which is here 5) in the quotient.

8^{thly}. Draw a line under the divisor, and multiplying the triple square (12) by 5, the figure last plac'd in the quotient, set the product (60) so under the said triple square, that units may stand under units, and tens under tens.

9^{thly}. Squaring the figure (5) last plac'd in the quotient, multiply its square (*viz.* 25) by the triple root (6), and place the product (150) so that units, in this, may stand under units in the said triple number.

10^{thly}. Subscribe the cube of (5) the figure last plac'd in the quotient, which will be 125; so that tens in this may stand under units in the former product.

11^{thly}. Then drawing a line, add the three numbers last plac'd together, and subtracting the sum 7635, (which is call'd the *ablatitum*) from the resolvend, set down the remainder (if any) in order underneath, as in common subtraction.

See the work :

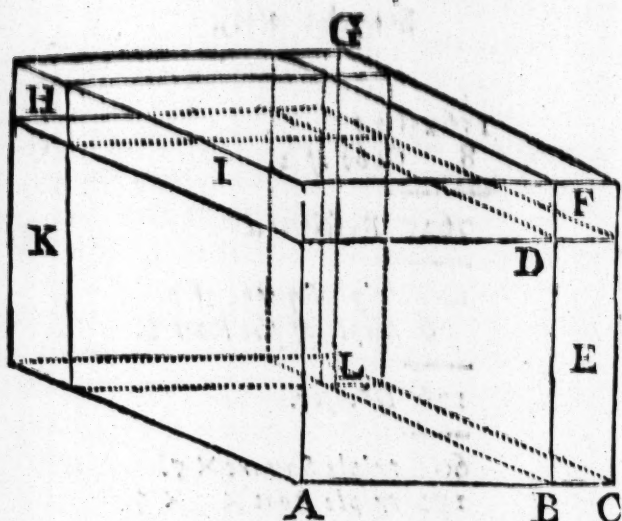
$$\begin{array}{r}
 15625 \overline{) 25} \\
 \underline{8} \quad \text{Cube of 2.} \\
 7625 \text{ Resolvend.} \\
 \underline{12} \quad \text{triple Square of 2.} \\
 \quad \underline{6} \quad \text{triple of the Root 2.} \\
 126 \text{ Divisor.} \\
 \underline{60} \quad \text{triple Square} \times 5. \\
 \underline{150} \quad \text{triple Root} \times 5 \times 5. \\
 \quad \underline{125} \quad \text{Cube of 5.} \\
 7625 \text{ Ablattitum.} \\
 \underline{\quad} \\
 \dots 0
 \end{array}$$

Note, If the given number hath more places, the next single cube must be brought down to the last remainder, for a new resolvend, and the work of the 4th, 5th, 6th, 7th, 8th, 9th, 10th, and 11th rules, must be repeated as often as you so form a new resolvend.

Note also, If the *ablattitum* is greater than the resolvend, the work is false, and must be rectify'd by placing a lesser figure in the quotient.

To account for these rules of extracting the cube root, we must again make use of a figure with literal references, as follows :

From



From whence it appears, That if the root $A. B.$ of the cube $A. D.$ be augmented by the magnitude $B. C.$ the cube is thereby enlarged by the addition of seven parallelepipeds, * *viz.* 1st, The little cube $G.$ whose root is the line $B. C.$ 2^d. The three parallelepipeds mark'd $I. K. E.$ which have the square of the root $A. B.$ for base, and the line $B. C.$ for height. 3^d. The three parallelepipeds mark'd $F. H. L.$ which have the square of the line $B. C.$ for base, and the root $A. B.$ for height.

In the operation therefore of the foregoing example, 15625 being divided into single cubes, it appears that the root will consist of two places: The nearest root then of the first cube 15, *viz.* 2, being plac'd in the quotient, where its value will be 20, from the place it stands in, and its cube 8000 subtracted from the given number 15625, the remainder 7625 will be equal to the seven parallelepipeds $G. I. K. E. F. H. L.$ the bases of three of which

* *Parallelepipeds is a Geometrical Term, signifying regular Figures, whose opposite Sides and Angles are equal.*

which, *viz.* *I. K. E.* being the square of the root *A. B.* three times the square of 20, *viz.* 1200 is set down under the said remainder, or resolvend; also three more of the said parallelepipeds, *viz.* *F. H. L.* having their height equal to the said root, three times 20, *viz.* 60, the sum of the three heights, is added to the said 1200, the sum of the bases of the other three, for a divisor; by which, dividing the resolvend, the number of times it goes, *viz.* 5, is the height of the three parallelepipeds *I. K. E.* and the root of the square of the bases of the other three parallelepipeds *F. H. L.* and also the root or side of the parallelepipedon, or little cube *G.* multiply therefore 1200 by 5, for the whole contents of the parallelepipeds *I. K. E.* *viz.* 6000; and squaring the said 5 for the bases of the three parallelepipeds *F. H. L.* multiply their three heights 60, by the said square of their bases, the product is 1500, their whole contents; under which setting down also the cube of 5, *viz.* 125, for the content of the parallelepipedon *G.* the sum of these three numbers I find to be just 7652, exactly equal to the resolvend; which shews the number given to be a perfect cube, whose root is exactly 25.

To make all more plain, see the work at length, with the vacant places fill'd with cyphers.

15625		20
8000	Cube A. D.	5
7625	Remainder or Resolvend	25 Root.
1200	Bases of Parallelepipeds I. K. E.	
60	Height of Parallelepipeds F. H. L.	
1260	Divisor.	
6000	Content of Parallelepipeds I. K. E.	
1500	Content of Parallelepipeds F. H. L.	
125	Content of Parallelepipedon G.	
7625	Total or Ablatitium.	

...0

PROOF.

P R O O F.

The proof of this extraction, is by multiplying the root found, first by it self, and that product again by the root, adding the remainder, if there be any, to the last product; thus,

$$\begin{array}{r}
 \text{multiply'd by} \quad 25 \text{ the Root} \\
 \hline
 125 \\
 50 \\
 \hline
 \text{gives} \quad 625 \\
 \text{again by} \quad 25 \text{ the Root,} \\
 \hline
 3125 \\
 1250 \\
 \hline
 \text{gives} \quad 15625
 \end{array}$$

To find the fractional part of the root,

Add to the number given, so many tenaries of cyphers, as you would have places of decimal parts in the root, and proceed by the foregoing rules. As suppose 865635 were given for an example.

EXAMPLE.

[225]

EXAMPLE.

865	635	000	000	000	(95,304 Rest.
729	635	—	—	—	1st. Resolvend.
136	3	—	—	—	
24	27	—	—	—	
24	57	—	—	—	Divisor.
121	5	—	—	—	
6	75	—	—	—	
	125	—	—	—	
128	375	—	—	—	Ablatitium.
8	260	000	—	—	2d. Resolvend.
2	707	5	—	—	
	2	85	—	—	
2	710	35	—	—	Divisor.
8	122	5	—	—	
	25	65	—	—	
		27	—	—	
8	148	177	—	—	Ablatitium.
	111	823	000	—	3d. Resolvend.
	272	462	7	—	
		28	59	—	
	272	491	29	—	Divisor.
	111	823	000	000	4th. Resolvend.
	27	246	270	—	
			285	90	
	27	246	555	90	Divisor.
	108	985	080	0	
		4	574	40	
				64	
	108	989	654	464	Ablatitium.
	2	833	345	516	Remainder.

In this example, there being three ternaries of cyphers, the root must have three decimal places.

Note, The divisor going no times in the third resolvend, the same figures are immediately brought down again; and with a new ternary of cyphers form the 4th resolvend.

To extract the Cube Root of a Vulgar Fraction.

The cube root of a vulgar fraction is found after the same manner as the square root, *viz.* by finding the root of the numerator given, for a new numerator, and the root of the denominator given, for a new denominator.

Also if the vulgar fraction be incommensurable, it must be reduc'd to a decimal, as directed in the square root, and then extracted.

F I N I S.

E R R A T A.

Page 80. *lin.* 1. for 4th, read 14th; and for 2d. read 3d. *pag.* 93. *lin.* 2. for 521. r. 5291. p. 115. l. 3. for 6893 Crowns, r. 2256 Crowns. p. 154. l. 7. for $\frac{1}{2}$, r. $\frac{7}{8}$. p. 155. l. 1. for $\frac{1}{2}$, r. $\frac{3}{4}$. p. 157. l. 3. for Quot. $31 \frac{1}{2}$, r. Quot. $31 \frac{3}{4}$. p. 184. l. 4. for 1000 miles, r. 100 miles.

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